## Analog Circuit Primer

This tutorial suggests some of the basic components of circuits and circuit diagrams, and presents some informal methods for analyzing circuits.

## 1 Fundamental Ideas

A circuit is a bundle of simultaneous equations. The equations come from the behavior of components and their arrangement.

For example, $v=i R$ is a common equation for circuits. If there is a voltage across a resistor, a current flows through it. The voltage does not "cause" the current, any more than the current causes the voltage (for if there is a current going through a resistor, a voltage will be across it). Rather, the relation $v=i R$ allows us to know one unknown (one variable), if we know the other.

The goal of circuit analysis is to "solve" circuits: to know all of the voltages and currents throughout the circuit, and how they change in time. And the nature of circuits is such that, with a little cleverness, you can always find as many equations in a circuit as there are unknowns.

Circuit diagrams and analysis are all about conventions. Within the conventions I describe, it is possible to describe equations one way. My conventions are common, but not universal, and when people use other conventions, some of the relations change.

For example, $v=i R$ is a relation between two directed quantities. The voltage across a resistor can be measured two ways (from left to right or from right to left), and the current may be said to go either of two ways (to the left or to the right). If you use $v=i R$ to solve for a current, say $5 m A$, how do you know which way those $5 m A$ are going? The answer is, by the convention behind the equation. $v=i R$ is only true if voltage and current are measured a certain way. Otherwise it would be $v=-i R$.

The conventions used here will be mentioned as they are used, and made explicit by diagrams accompanying the text.

There are many methods for solving circuits. Some techniques are very algorithmic (PSPICE uses these), others are very intuitive (Gill uses these). This document approaches circuits as puzzles. Moreover, this document does not cover digital circuits and logic. Another document, and manner of thinking, is required for that subject.

The fundamental variables involved in circuits at this level are simply voltage, current, and resistance.

Voltage : Physically, voltage is the integral of an electromagnetic field, just as the potential energy of physical height is the integral of gravitational force. In electrical engineering, voltage is often the variable we are most concerned with, in part because it is easy to measure. Voltage is always relative: the voltage "at a point" is a nonsensical concept, although one often used as shorthand. Rather, voltage must always be described as existing between two points. Note that physical height must be measured the same way: we measure our mountains from sea-level, our people from head to foot.

Current : Current is the flow of charge (electrons and the absence of electrons). Current is measured through a point, or a component, or a wire. Electronic components generally do not store net charge, so it is fair to say that the current entering an electonic component is the same as the current leaving it.

Resistance : Physically, resistance is a characteristic of things that waste energy. Voltage across any electronic component represents stored energy, but across a resistor it will produce a current through the resistor, and heat. The resistance of any well-behaved material is porportional to its length and inversely porportional to its width: the more material to travel though, the higher the resistance; the more paths available to travel, the lower the resistance.

One necessary part of solving a circuit is defining its unknowns. This is done by giving names to various voltages and currents, and along with the names, directions. When defining a voltage between two points, label one with $a+$, the other with $a-$ : the voltage then represents the difference between the potential at those two points. For a current, define a direction for the current flow, by drawing an arrow $(\rightarrow)$. Your chosen direction may be "wrong" - current may actually be flowing the other direction. You would discover this by getting a negative result. Negative current in one direction is positive current in the other direction; a negative voltage difference measured one way is a positive voltage difference measured the other way.


## 2 Elementary Components and Arrangements

Every electronic component, or combination of components, is defined by the constraint it places on the voltage across it and the current through it. Below are some common elements.

## Voltage Sources :



## Current Sources :



## Resistors :



Voltage sources specify a voltage difference, but not a current. They are normally labelled with the + and - direction on the component and the voltage difference next to the component. Any amount of current needed may go through a voltage source, as specified by the other constraints of a circuit.

$$
v=V
$$

Current sources specify the current going through a component, but any amount of voltage may be across a current source. Current sources are labelled with the direction of current $(\leftarrow$ or $\rightarrow$ ) and the amount of current.

$$
i=I
$$

Resistors specify a relation between the voltage and current: $V_{R}=I_{R} R$. The equation assumes the convention of Spurk's law: "current always travels through resistors in the direction of voltage fall" (current flows into the + terminal on the resistor). In other words, if you specify the direction of current to flow into the + terminal on a resistor, you will never find that the current is negative unless the voltage is as well.

$$
V_{R}=I_{R} R
$$

Circuit components are combined in infinite unique ways. However, often pieces of larger circuits can be described as having common structures, and those common structures effect how the sub-circuits behave. Below are the two most common structures.

## Series Circuits :



## Parallel Circuits :



A series construction means that the same current will flow through both components or subcircuits, although the voltage across them will be different. However, for reasons explained shortly, the combined voltage across both components must be equal to the voltage across the whole circuit.

$$
i=I_{A}=I_{B} ; v=V_{A}+V_{B}
$$

A parallel construction specifies that the voltage across the two subcircuits is the same. Additionally, the sum of the currents flowing into each subcircuit equals the current flowing into the whole circuit (the current splits).

$$
v=V_{A}=V_{B} ; i=I_{A}+I_{B}
$$

## Solving a Circuit

Solve for the voltage $V_{\text {out }}$ in the following circuit.


Notice the structure of the circuit. The left side has two components in parallel, fed by some voltage ( $V_{\text {out }}$ ) and some current $\left(I_{1}\right)$. Taking the left side as a single component reveals that the entire circuit is two components in series, where the solution $\left(V_{\text {out }}\right)$


Start by labelling the circuit with voltages and currents, as below. You may label additional voltages and currents, if you wish, but it will soon become apparent that, for example, the voltage across the $2 \Omega$ and $3 \Omega$ resistors are equal (see ??).


Now write down the constraints of the circuit that relate our unknowns:

$$
\begin{array}{r}
V_{1}=I_{1}(1 k \Omega) \\
I_{1}=I_{2}+I_{3} \\
V_{\text {out }}=I_{2}(2 k \Omega) \\
V_{\text {out }}=I_{3}(3 k \Omega) \\
5 V=V_{1}+V_{\text {out }} \tag{5}
\end{array}
$$

5 Equations, 5 Unknowns. Solve the equations to get:

$$
\begin{array}{r}
V_{1}=25 / 11 \mathrm{~V} \\
I_{1}=1 / 1100 \Omega \\
I_{2}=3 / 2200 \Omega \\
I_{3}=1 / 440 \Omega \\
V_{\text {out }}=30 / 11 \mathrm{~V} \tag{10}
\end{array}
$$



Now travel around the circuit and see how the equations have found their solution.

## 3 A Little Formalization

Kirchhoff's Laws are the basis for solving any circuit, and are closely tied to the definitions of voltage and current.

## Kirchhoff's Voltage Law :

Kirchhoff's Voltage Law (KVL) says that the voltages around any circuit loop sums to $0 .$. .
 as long as you count them according to convention: voltage rises count positively; voltage falls count negatively. So in the accompanying circuit, counting around the outer loop, $\left(-V_{A}\right)+\left(-V_{B}\right)+V_{D}+V_{C}=0$; but also $-\left(V_{A}\right)-\left(-V_{X}\right)+V_{C}=0$. KVL comes directly from the conservative nature of voltage.

$$
\sum_{i} V_{i}=0
$$

## Kirchoff's Current Law :



Kirchhoff's Current Law (KCL) says that the currents travelling into (by convention) a node, or intersection of wires, sum to 0 . In the circuit, $I_{A}+I_{C}+I_{D}-I_{B}=0$. In other words, all the current entering a node has to leave it.

$$
\sum_{i} I_{i}=0
$$

Some purely conceptual circuit elements that you might already know about are open circuits and closed circuits (or short-circuits).

## Open Circuits :



Open circuits are unconnected wires from different points in the circuit: there will often be a voltage between them, but no current.

$$
i=0 A
$$

Short Circuits :


Short circuits are wires; current will flow through them, but voltage cannot develop across them.

$$
v=0 \mathrm{~V}
$$

## 4 Equivalent Circuits

Another, related, method for solving circuits, is by simplifying the circuit. For linear circuits, many large circuit pieces can be replaced with simplier subcircuits which act the same way, when judged from outside the subcircuit. In otherwords, the simplified subcircuits place the same constraints on voltage and current as the more complicated subcircuits. All of the circuit equivalences below are easy to prove.

## Rearranging Series Circuits :


$\bar{T}$ The order of elements in a simple series construction has no effect on the behavior of the circuit.

$$
V=V_{A}+V_{B}
$$

## Rearranging Parallel Circuits :


$\overline{\text { Similarly, the order of components in parallel construction is irrelevant, and does not }}$ even necessarily have a real-world basis.

$$
I=I_{A}+I_{B}
$$

## Series Resistors :



Any number of resistors in series acts the same as one resistor with the sum of all their resistance values. This makes sense, analytically and intuitively. Analytically, the same current must flow through all of the resistors, producing a total current drop of $V_{\text {total }}=I\left(\sum R_{i}\right)$. This is a generalized voltage divider.

$$
R_{\text {Combo }}=\sum_{i=A}^{Z} R_{i}
$$

## Parallel Resistors :



Resistors in parallel act like a single resistor, with a resistance smaller than the smallest parallel resistor. This is a generalized current divider.

$$
\frac{1}{R_{\text {Combo }}}=\sum_{i=A}^{Z} \frac{1}{R_{i}}
$$

## Series Voltages :



Voltage sources in series act like a single voltage source.

$$
V_{\text {Combo }}=V_{A}+V_{B}
$$

## Parallel Currents :



Current sources in parallel act like a single current source.

$$
I_{\text {Combo }}=I_{A}+I_{B}
$$

## Thevinin/Norton Equivalents :



In fact, any resistive circuit can be simplified to a Thevinin or Norton equivalent, with enough simplifying and cleverness, or with more formalized methods not discussed here.

$$
V=I R
$$



Start by rearranging the circuit to make it clearer:


The two parallel resistors can be combined.


The left hand current source and resistor have a Thevinin equivalent (a voltage source and a resistor).


Now the voltage source and resistors can be combined by summing, to get the simplest result.


By Kirchhoff's voltage law, the voltage across the resistor is 5 V , so the current through the resistor is $5 \mathrm{~V} / 3.67 \mathrm{k} \Omega=1.36 \mathrm{~mA}$.

## 5 Capacitors and Inductors

There are many ways to look at the behavior of capacitors and inductors. However, they all derive from the same fundamental equations, as shown below.

## Capacitors :



Capacitors store electric fields (equal amounts of positive and negative charge). As current flows through them, opposite charges build up on two parallel plates within the capacitors, producing an increasing voltage. If such a charged capacitor is then placed across a resistor, it will act as a voltage source until the voltage across the capacitor decreases to zero. Capacitors themselves never waste energy.

$$
i=C \frac{d v}{d t} ; v=\frac{1}{C} \int i d t
$$

## Inductors :



Inductors store magnetic field, which in some ways is like storing current. Voltage across an inductor causes the current through an inductor to increase.

$$
v=L \frac{d i}{d t} ; i=\frac{1}{L} \int v d t
$$

Over time, if nothing changes in a circuit with a capacitor, the voltage across the capacitor will approach a certain level, as the current through the capacitor goes to zero (as the constraints on the circuit change). Similarly, an inductor over time will approach having a certain current through it and the voltage across it will go to zero.

$$
\begin{array}{rll} 
& \text { Capacitors } & \text { Indcutors } \\
\text { Equations: } & i=C \frac{d v}{d t} & v=L \frac{d i}{d t} \\
\text { Resists change in: } & \text { Voltage } & \text { Current } \\
\text { Tries to act like: } & \text { Open Circuit } & \text { Short Circuit }
\end{array}
$$

There exists another way to look at capacitors and inductors, as a kind of resistor with a complex resistance, but that is beyond the scope of this tutorial.

You might wonder, "How did all these beautiful relations get there?" In part, it is the product of human ingenuity: we have developed powerful ways of looking at circuits, and
these elegant relations come out of those mechanics because we put them there without realizing it. The other reason is that circuits are metaphors for some of the principles which underly all engineering and physics: across and through, equivalency and system structures, connections and relations. That beauty is the beauty of the world, but like a polished mirror, circuits reflects it better than most disciplines.

## 6 Op-Amps, Transistors, and Diodes

While all of the idealizations used thus far are simplifications, the simplifications in this section will be more gross here than elsewhere. A more complete description of the behavior of op-amps, transistors, and diodes can be found in The Art of Electronics by Paul Horowitz and Winfield Hill.

## Operational Amplifiers :

Operational Amplifiers adjust their output according to the voltage difference between their $+i n$ and $-i n$ inputs. Depending on the relationship between the output and the input, the op-amp is said to be in positive or negative feedback. In positive feedback, the op-amp quickly sets its output as high or low as possible. In negative feedback, the op-amp quickly adjusts its output so as to make its inputs equal. However, the op-amp's output is limited to a range slightly smaller than the one between $V_{+}$and $V_{-}$.

$$
V_{\text {out }}=A \int V_{\text {in }}, \text { for large } A
$$

## Diodes :



Diodes only allow current to flow in one direction through them. However, for normal ranges of voltage, when current flows through them, there will be an approximately .7 V drop across them. The exceptions are special types of diodes (zener diodes and tunnel diodes) which work differently, and when a very large voltage is applied in the "wrong" direction, at which point the diode temporarily "breaks down".

$$
I \geq 0 \text {; for } I>10 \mathrm{~mA}, V \approx .7 \mathrm{~V}
$$

## Transistors :



Transistors are often used like voltage controlled switches. They have three terminals: a base, a collector, and an emitter. When a sufficiently high voltage is applied to the base (around .7 V ), current can flow almost uninhibited between the collector and emitter. The voltage on the emitter is determined by the base voltage ( .7 V less, when the transistor is conducting).

$$
V_{B E} \approx .7 V ; I_{C}=\beta I_{B} \approx I_{E}, \text { for large } \beta
$$

