Effects of parameter changes in weighted averaging

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1 Introduction

Electric or magnetic signals induced by electro-chemical activity in peripheral nerves are typically orders of magnitude lower than other biological signals, external environmental noise and instrumental noise. As a means to reconstruct these low-amplitude Evoked Responses (ERs) from repeated measurements of stimulated nerves, a conventional ensemble averaging (EA) technique is often applied. However, the classical hypothesis of an evoked signal embedded in stationary and uncorrelated noise usually does not apply to real measurements of ERs. The weighted averaging routine [1], which uses weights for each epoch inversely proportional to the variance, has thus previously been introduced as an improvement of the EA routine. Recently, weighted averaging was further refined by considering the covariance between the epochs [2,3,4], thereby introducing a host of parameters that may have an effect on the final result. The present work is based on the weighted averaging assuming correlated noise (WA).

The obvious advantage of WA in comparison to EA lies in the ability to reduce artefacts to a minimum in the processed signal. Artefacts are often generated by interfering biomagnetic sources such as the heart or muscles, but can also arise from external sources, such as switching near-by electric engines, moving cars, elevators etc.

The aim of this work is to clarify the dependence of certain parameters on the WA, in order to optimise the application and to obtain a better understanding of the method. The parameters examined include the response to various components in the data series, such as heart beat and gaussian noise, minimisation of underestimation, block processing and Estimated Evoked Response Window (EERW) variations.

2 Weighted averaging

The basic principles of the WA routine will be sketched briefly here; the reader is referred to [2] for a more thorough presentation of WA. The basic task is to find optimal weights that suppress noise and artefacts as effectively as possible, whilst maintaining the ER unperturbed. The measured data in a multichannel arrangement is assumed to contain signal and noise, as

$$y_{i,k}(t) = s_k(t) + n_{i,k}(t)$$

where index i represent the epoch, defined as fixed time intervals between two consecutive stimuli, t the time and k the channel. Assuming the nerves to be invariant to stimuli in time, the epoch-independent $s_k(t)$ will be time locked to each stimulus. The nerve is stimulated M_T times. The optimal weights are obtained by solving the following equation [2]

$$\begin{pmatrix} \sum_{k=1}^{K}\sigma_{1,k}^2 & \sum_{k=1}^{K}\mu_{12,k} & \cdots & \sum_{k=1}^{K}\mu_{1M,k} & -1 \\ \sum_{k=1}^{K}\mu_{21,k} & \sum_{k=1}^{K}\sigma_{2,k}^2 & & & -1 \\ \vdots & & \ddots & & \vdots \\ \sum_{k=1}^{K}\mu_{M1,k} & & & \sum_{k=1}^{K}\sigma_{M,k}^2 & -1 \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} w_{1,opt} \\ w_{2,opt} \\ \vdots \\ w_{M,opt} \\ mmse \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

where K is the number of channels, M is the so-called blocksize, i.e. the number of epochs for which the ER is estimated, mmse is the minimal mean square error of the estimated signal and $\mu_{ij,k}$ is the empirical covariance between the i^{th} and the j^{th} epoch of the k^{th} channel, given by

$$\mu_{ij,k} = \frac{1}{N-1}\sum_{t=1}^{N}\left(y_{i,k}(t) - \bar{y}_{i,k}\right)\left(y_{j,k}(t) - \bar{y}_{j,k}\right)$$

where N is the number of time samples in one epoch and $\bar{y}_{i,k}$ the average over time of $y_{i,k}(t)$. Due to the fact that the covariance matrix is singular for $M \geq N$, the weights are estimated blockwise, i.e. the total number of epochs, M_T, is divided into blocks with $M \leq N$ [2]. The estimated evoked response (EER) of each block is given by weighted sum of the respective epochs, as

$$\hat{s}_{k,b}(t) = \sum_{i=1}^{M} w_{i,opt}\, y_{i,k}(t)$$

where b is the block number. These block estimates are subsequently processed further.

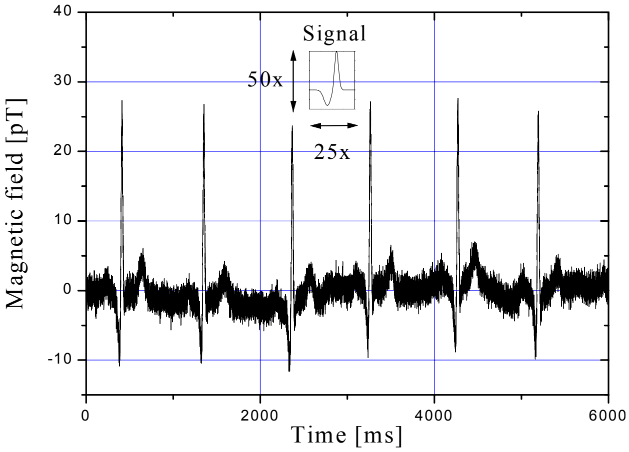

Figure 1: *The simulated data series is a superposition of gaussian noise, baseline drift, heart signal and ER.*

In order to minimise the effect of underestimation[1], the last EER can be subtracted from each epoch before estimating the covariance matrix. The corresponding weights are then applied to the original epochs to give a new estimate. Since these weights are not optimal to the original signal, the subtraction is restricted to a window interval containing the ER. This is the EERW, and within this window underestimation will be minimised. Samples not located in this interval are part of the so-called residual signal. These two temporal regions will be investigated separately.

3 Method

The data investigated is entirely simulated and consist of a single channel time series (K=1) with the weighted sum of 3 different components, namely heart beat (P-QRS-T-waveform, weight 250), gaussian noise (weight 20), and a repeated biphasic evoked response waveform (weight 1) (see figure 1). This sum is called the full data. The amplitudes and waveforms of the signals are chosen to reflect a realistic measurement of the activity evoked in a peripheral nerve in the lower cervical or the upper lumbar region.

The advantage of this numeric approach is that it facilitates the investigation of the influence of each signal separately, and it renders possible the evaluation of the merit of the outcome.

The reconstruction merit of the estimate is defined as

$$\xi = \frac{\sigma_s}{\sigma_{\hat{s}-s}}$$

where σ_s is the standard deviation of the pure ER in the EER window and $\sigma_{\hat{s}-s}$ is the standard deviation of the difference between the estimated signal and the original signal. This can also be interpreted as the signal-to-noise ratio.

4 Results

4.1 Response to single signal components

As a means to avoid $M_T \geq N$, the epochs are divided into blocks of blocksizes less than N, and the block signals are then processed further, either by use of EA or WA. The block approach also enhances processing time. For M close to N, the variance of the estimated signal is erratic and close to zero.

In figure 2 the dependence of ξ on blocksize is examined for single components of the data. Only the result within the EER window is considered here. The separate components are examined by adding 4500 consecutive time-stamped ERs of 190 samples to a long time series containing a single component, e.g. gaussian noise, and processing the data with the WA routine to estimate the ER. Looking at heartbeats only, figure 2 shows an efficient recovery of ER at lower blocksizes. A well-defined maximum is present at a point where the disturbing artefacts from the heartbeat are suppressed as efficiently as possible to give ξ over 100 at blocksize 6. As comparison, the EA of the time series (dominated by heart) gives ξ around 2. The smooth low-frequency artefact of the heart can thus be effectively compensated for by the WA routine.

The ensemble average is, as indicated, heavily influenced by the heart beats, and for full data, ξ is over 35% higher for WA than EA. This figure de-

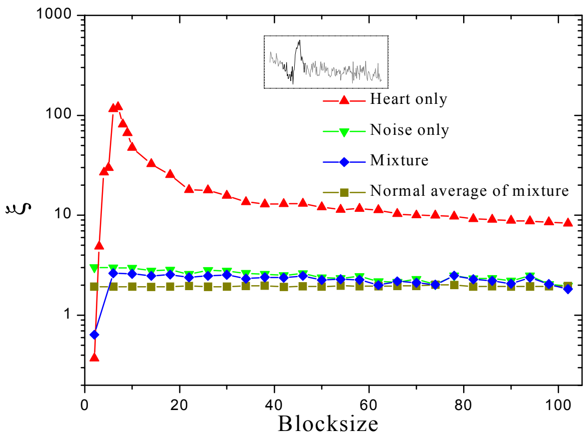

Figure 2: *The signal-to-noise ratio as function of blocksize. The EER window is indicated in the inset.*

[1] Lütkenhöhner et al. showed in [1] that due to the fact that the weights were estimated on the basis of y and not on the basis on n, the resulting estimate ŝ is smaller than s. This effect is called ‘underestimation’

pends strongly on the waveform and amplitude of the MCG. Figure 2 shows that WA of full data is limited upwards by WA of gaussian noise and downwards by EA. The WA has therefore no advantage to EA with regard to gaussian noise, but large advantage to EA with regard to heart beats, i.e. to signal components with large covariances between the epochs.

4.2 Minimisation of underestimation

In determining the empirical covariance, the signal will inevitably contribute to the covariance. The estimated signal is thus prone to underestimation of the ER with up to 100% [1]. However, an iterative subtraction of the best possible EER in the EER window will ideally lead to convergence towards a better estimate of the ER (IWA = iterative WA).

If the original ER, uninfluenced by noise, could be uncovered and subtracted before the optimal weights were estimated, the effect of underestimation would be eliminated. Now, the ER from peripheral nerve measurements seen as an isolated event, will most probably convey a smooth waveform. Thus, an iterative subtraction of a lowpass filtered EER without significant phase changes within the passband, is very likely to perform better than unfiltered.

Figure 3 confirms this statement. It shows the reconstruction merit as function of iterations calculated inside and outside the EER window, respectively. The subtraction data was filtered for the first 10 iterations, and then, continuing the algorithm, the non-filtered last iteration EER was subtracted for another 10 iterations. Within the EER window, the filtered subtraction converges towards ξ about 30% higher than unfiltered subtraction, thereby improving the outcome considerably. The extent of this

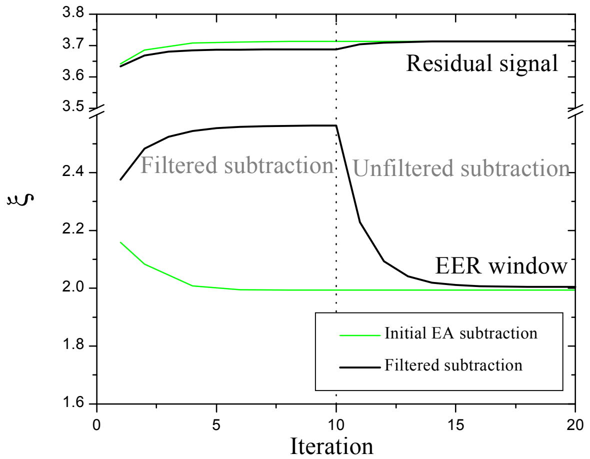

Figure 3: *ξ as a function of iterations. The graph is left-right divided into a filtered and non-filtered subtraction region.*

improvement is dependent on blocksize and filter parameters.

As observed, as soon as filtering is discontinued, the EER immediately starts to converge toward the curve of non-filtered subtraction. This effect is yet unexplained. However, the trend is evident and a platform for improvement has been unveiled.

4.3 Block processing

In this section, different approaches to block processing are investigated. Figure 4 show the merit of the signal with different variations of block processing as a function of blocksize. An EA and a WA assuming uncorrelated noise (UWA, from [1]) of the full data are shown for comparison.

IWA strongly attenuates the effect of artefacts outside the EER window, both in comparison to EA and UWA (see figure 4). However, inside the EERW, the efficiency of the WA is improved by IWA but still challenged by the UWA. We thus find that in this case the UWA is comparable to IWA in the EER window. In order to gain insight into the underlying mechanisms of this the main problem by

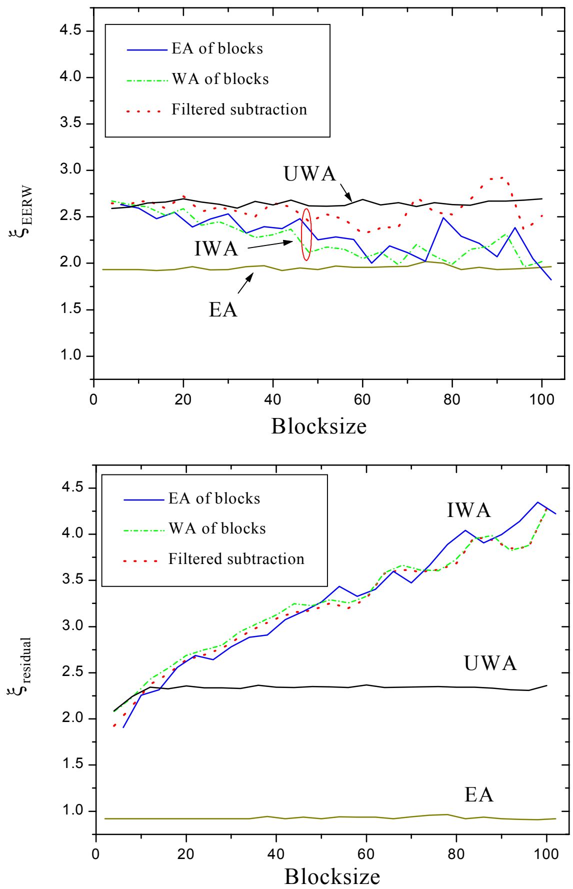

Figure 4: *The effect of different types of block processing on the signal within the EER window (top) and the residual signal (bottom). No significant differences in merit are seen, even though filtered subtraction tends to improve the result somewhat.*

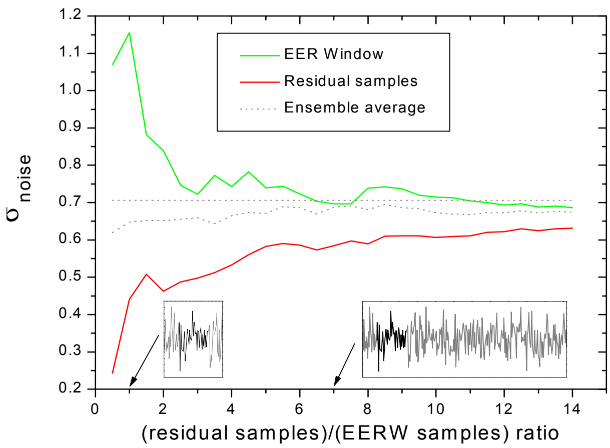

Figure 5: *Here singly Gaussian noise is processed. The variance of noise within the EER window and outside is shown as function of the ratio between the number of samples in the respective regions.*

WA, one must note that the subtraction of the last estimate in the EER window perturbs the signal in the window, but not outside. Although the perturbation is slight the underestimation can as mentioned reach up to 100% under certain conditions. The optimal weight is estimated for this perturbed epoch, but applied to the original. Hence, the weights are no longer optimal for the signal in the EER window but (depending on the EER window ratio) only slightly displaced from the optimum outside. Therefore, random noise will be of larger amplitude inside the EER window whilst being lower on the outside. This also explains the improvement by filtered subtraction, which is also evident in figure 4, since subtraction signal is smoother, leaving more noise to compensate for to the WA.

The results show that there is no significant improvement in ξ when the blocks are averaged by weights, since the non-stationarity of the noise and artefacts has been suppressed within the block. In conclusion, EA is sufficient for block processing.

4.4 EER window

The ratio between the number of samples in the residual signal and in the EER window will have an effect on the result, especially when this ratio is low. This is understood by considering the limiting example in which the epoch is limited to the EER window. The optimal weights are then estimated for epochs in which all samples differ from the original epochs they are applied to, thus leading to erroneous estimations of the ER. A simulation was performed using only gaussian noise and no ER in order to investigate how the WA would respond in the ideal situation where all components of s are estimated and subtracted. Figure 5 shows that as an increasing number of samples are used for calculation of the empirical covariance between two epochs, the better the noise and artefact compensation in the EER window, assuming the EER window to be constant. It is observed that one should as a minimum have a ratio of 3 to obtain good results. Both signal regions converge toward the EA of the gaussian noise, indicating that a large ratio is to be preferred for large signal to noise ratio.

5 Conclusion

In this work we have given an overview on the different parameters of the weighted averaging method and their influence on the result. We found that in the presents of large correlated perturbations, WA requires the considerations of the covariance. To avoid underestimation of the signal, an iterative approach of WA is recommendable. Furthermore, it may be advantageous to filter the iterative subtraction signal in an interval enveloping the evoked response. This interval should be as narrow as possible.

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