Corrections for Initial Masses of Large Fireballs from the Canada Network

Maria I. Gritsevich* and Vladimir P. Stulov[1]
Institute of Mechanics, Moscow Lomonosov State University, Michurinskii Ave., 1, Moscow,119192, Russia

[Abstract] Earlier the initial mass of meteoric bodies was defined under the so-called photometric formula by integration of luminosity along a visible site of a trajectory. On the other hand, the mass of a meteoric body characterizes height and intensity of braking of a meteor in an atmosphere. In a number of works, the essential divergence of masses received by these two ways was marked, on an example of fireballs from the European network and from the Prairie network, USA. The photometric mass exceeds the mass defined on intensity of braking on the order and more practically always. Explanations to this circumstance are published. One of them consists in the assumption that a plenty of equivalent fragments moves but not a single body. The plenty is braked as a single fragment, and shone as set of fragments, i.e. is much brighter than a single fragment. In this work, the pre-atmospheric mass is defined by selection of the parameters describing braking of a meteor along all visible site of a trajectory. Results for fireballs from the Canadian network again confirm an inconsistency of the photometric approach.

Nomenclature

A	=	factor of the body shape
H^*	=	effective enthalpy of destruction
h	=	height
M	=	mass
S_e	=	middle section area
V	=	velocity
α	=	ballistic parameter
β	=	mass loss parameter

I. Introduction

The question on a way of determination of initial masses of meteoric bodies is discussed in the literature for a long time. The detailed review and the analysis deserve the separate edition. Therefore here we shall result only some references.

At processing observations the concept of photometric mass of a meteoric body is widely used

$$M_{ph} = -\int_{t_1}^{t} \frac{I}{\tau V^2} dt$$

Here I is luminosity of a meteor, V is speed, τ is the luminous efficiency due to ablation. It is considered that integration on all visible sector of an atmospheric trajectory, i.e. from time of going out t = t_1 till time of occurrence of a meteor t = t_0 gives value of preatmospheric mass of a meteoric body.

The analysis of fireballs from to the European network is given in one of the first works on this subject (Ceplecha, 1978). Besides the photometric mass, values of mass in a final point the trajectories defined on observable braking are resulted. The calculations stated in last section of our work show that photometric data strongly overestimate values of preatmospheric masses.

Researchers have met similar difficulties at the analysis of fireballs from the Prairie network in USA (McCrosky *et al*., 1971), in particular, at studying an atmospheric trajectory of the Lost City meteorite. Comparison of full photometric mass to total mass of the found fragments testifies to unreasonably big ablation during movement in the atmosphere. Indirect researches of mass ablation on measurements of space beams traces in meteorites, and also radioisotope methods show more moderate mass loss owing to ablation. To reduce this divergence, authors changed τ in the photometric formula. To have reasonable values of the initial mass of meteoroid Lost City (about 50-100 kg), τ should be increased of in eight times.

The similar situation has arisen at studying of the Beneshov bolide, fixed on May 7, 1991 by the stations in Czech Republic, which are the part of the European network. The estimation of the initial mass on observable braking (Barry and Stulov, 2003), and also by means of a method a gross-fragmentation (Ceplecha *et al*., 1993) has given value no more than 100 kg. On the other hand, calculations under the photometric formula, and also by a method of radiating radius show that the initial mass of the bolide is 4000-13000 kg (Borovicka *et al*., 1998). Such significant difference of estimations retains till now.

II. The Description of Atmospheric Trajectories

The solution of the meteoric physics equations

$$m = \exp\left[-\frac{\beta}{1-\mu}\left(1-v^2\right)\right], \quad y = \ln\alpha + \beta - \ln\frac{\Delta}{2}, \quad \Delta = \overline{\mathrm{Ei}}(\beta) - \overline{\mathrm{Ei}}(\beta v^2), \quad \overline{\mathrm{Ei}}(x) = \int_{-\infty}^{x}\frac{e^t\,dt}{t} \tag{1}$$

shows that the trajectory depends on two dimensionless parameters α and β (Stulov *et al*., 1995)

$$\alpha = \frac{1}{2}c_d\,\frac{\rho_0 h_0 S_e}{M_e \sin\gamma}, \quad \beta = (1-\mu)\frac{c_h V_e^2}{2c_d H^*}. \tag{2}$$

Here an angle γ, drag c_d and heat transfer c_h coefficients and also effective enthalpy of destruction H^* are constant. Velocity V_e, body mass M_e and the middle section area S_e on entry into the atmosphere (an index *«e»*), and also height of the homogeneous atmosphere h_0 and density of gas on the sea level ρ_0 are in formulas (2). The basic dimensionless variables are: height $y = h/h_0$, velocity $v = V/V_e$ and body mass $m = M/M_e$. Parameter α characterizes intensity of braking as it is proportional to the ratio of the atmospheric column mass with cross-section S_e along the trajectory to body mass. Parameter β is proportional to the ratio of a portion of kinetic energy of a unit body mass acting on the body in the form of heat, to effective enthalpy of destruction.

Value of parameter $\mu = \log_m s$ ($s = S/S_e$ is the middle section) characterizes a possible role of rotation in flight: $\mu = 0$ when rotation is absent, $\mu = 2/3$ when ablation occurs uniformly on all surface due to rotation, so the factor of the body shape $A = S/W^{2/3}$ (W is volume) is constant.

The integral exponent Ei(x) in the analytical solution for trajectories complicates the further calculations. Therefore at the limited values of the mass loss parameter β, the solution is replaced to more simple approached expressions (Stulov *et al*., 1995)

$$Y(v,\beta) = y(v,\alpha,\beta) - \ln\alpha = -\ln(-\ln v) + 0.83\beta(1-v) \tag{3}$$

Comparison of functions (1) and (3) is resulted on Fig. 1 for β = 1, 2, 3. In the solution (1), the integral exponent Ei(x) was calculated as an expansion at the limited values of argument

(Janke *et al*., 1964). Comparison shows that the approached representation of the solution by the function (3) is possible to use with sufficient accuracy in a range $0 \le \beta \le 3$.

There is a natural restriction of the specified approximation. Function (3) has a point of inflection at $v = e^{-1}$. The derivative of function (3) in this point is equal $Y'(e^{-1},\beta) = e - 0.83\beta$. In other words, at $\beta = e/0.83$, the point of an excess turns to a minimax so at $\beta > e/0.83 = 3.275$ function (3) has a minimum and a maximum, i.e. it becomes unsuitable for the description of the trajectory within the limits of considered model.

III. Method of the Least Squares

The purpose of this work is determination of parameters of meteoric bodies according to observations of the Canadian camera network (Halliday *et al*., 1996). As it was already marked, exact (1) and the approached (3) expressions for atmospheric trajectories depend on two dimensionless parameters (2) describing braking and ablation of a meteoric body. Therefore at selection of these parameters the main attention should be given those sectors of the trajectory where braking and ablation are precisely enough expressed. On the other hand, preliminary processing of observant data to choose the priority sectors would complicate research, would make it insufficiently objective. The alternative is solved by use of a trajectory (3) in the following form

$$\alpha \exp(-y) + \ln v \exp[-0.83\beta(1-v)] = 0 \tag{4}$$

Required parameters α and β are defined by values on which the minimal value of following expression is reached

$$Q_3(\alpha,\beta) = \sum_{i=1}^{n} F^2(y_i, v_i, \alpha, \beta) \tag{5}$$

Here $F(y_i, v_i, \alpha, \beta)$ is the left-hand side of (4), and y_i, v_i are data of observations. The similar variant of the least squares method was used earlier in works (Stulov, 2000; Barry and Stulov, 2003) where analytical expressions for trajectories in view of consecutive fragmentation were used as trial functions except for (4).

Minimum of function $Q_3(\alpha, \beta)$ is defined by the rules

$$\left.\frac{\partial Q_3}{\partial \alpha}\right|_{\beta} = 0, \quad \left.\frac{\partial Q_3}{\partial \beta}\right|_{\alpha} = 0 \tag{6}$$

The first part of (6) gives obvious expression for parameter α

$$\alpha = \frac{-\sum_{i=1}^{n} \exp[-0.83\beta(1-v_i) - y_i]\ln v_i}{\sum_{i=1}^{n} \exp(-2y_i)}, \tag{7}$$

and the second one allows to work out the transcendental equation for one unknown parameter β

$$\sum_{i=1}^{n} \{\alpha \exp(-y_i) + \ln v_i \exp[-0.83\beta(1-v_i)]\} \ln v_i (1-v_i) \exp[-0.83\beta(1-v_i)] = 0, \tag{8}$$

where α is given by the formula (7).

The approximate solution of (8) can be found as follows. We shall put $x = b - 1 = \exp(-0.83\beta)-1$ and spread out the left part of (8) in expansion on x, keeping linear and square-law composed. Solving a corresponding quadratic equation and choosing its root getting on half-interval [-1, 0], we shall receive required value of mass loss parameter $\beta = -0.83^{-1}\ln(x+1)$.

As it was already marked, application of trial function in the form of (4) allows to use all available observant basis (y_i, v_i), not rejecting in advance points with small braking and ablation. These points take place in initial sector of a trajectory and correspond rather to great values y_i which give in (4) the exponential small (smaller) contribution.

The exact solution of the equation (8) is found numerically by the method of Newton. As initial approach, value $b_0 = 1$ or a root of the square-law approach described above is taken. Usually, it does not lead to substantial growth of iteration number.

IV. Preatmospheric Masses of Meteoric Bodies

Calculations of parameters α, β for a number of fireballs from the Canadian camera network (Halliday *et al.*, 1996) have been lead. As the basic table in the equation (8), all observational points were accepted in all cases. Simultaneously, preatmospheric mass M_e was determined on values of parameter α

$$M_e = \left(\frac{1}{2} c_d \frac{\rho_0 h_0 A}{\rho_m^{2/3}} \frac{1}{\alpha \sin \gamma} \right)^3 \tag{9}$$

At calculation of M_e, the following numerical values of parameters were accepted: $\rho_0 = 1.29 \cdot 10^{-3}$ g/cm^3, $h_0 = 7.16$ km. As well as in work Halliday *et al.* (1996), it was considered that a meteoroid has the form of a rectangular parallelepiped (a brick-like shape) with the edges $2L$, $3L$ and $5L$ and the face plane 3x5, so the factor of the body shape $A = 15L^2/(30L^3)^{2/3} = 1.5536$; besides $\rho_m = 3.5$ g/cm^3, $c_d = 1$ (Halliday *et al.*, 1996).

Observable trajectories are curvilinear, so sinγ changes along trajectories. The value of sinγ was calculated under the formula

$$\frac{dh}{dt} = -V \sin \gamma \tag{10}$$

using the central differences for internal points. In the formula (9), the average arithmetic value of sinγ on all observable points was used.

Results of our calculations for 22 fireballs from the Canadian network are given in Table 1. Also the values of preatmospheric masses MI (Halliday *et al.*, 1996, Table 4) are given in Table 1. The basic part of values MI are the photometric masses m_p (see Table 6 of Halliday *et al.*, (1996)) in most cases.

Examples of approximation of observant trajectories with use of the found values α, β are shown on Fig. 2 for fireballs 018 and 567. As one would expect, the best approximation concerns to area of the developed braking ($v \leq 0.9$).

The basic result of the calculations shown in Table 1 consists in essential difference of the initial masses determined by braking on all observable site of their trajectory, on the one hand, and the masses received on the basis of intensity of fireball luminescence (photometric masses), on the other hand.

The Canadian authors had been undertook efforts on updating factor of the luminous efficiency τ depending on velocity. These efforts are represented us hopeless as the reason lays not in imperfection of the known photometric formula, and in illegality its application in considered cases.

Last statement is illustrated in Table 2 for four fireballs. The first 5 columns are taken from the work (Halliday *et al.*, 1996, Table 6). Here h_b, h_{ml}, h_t are heights of the beginning of shone sector of a trajectory, the maximal luminosity and the termination of shone sector. Values L and R_{eff} correspond to the characteristic sizes of meteoroids according to the initial masses M_e.

$$L = \left(\frac{M_e}{30\rho_m} \right)^{1/3}, \quad R_{eff} = \left(\frac{30L^3}{4\pi/3} \right)^{1/3} \tag{11}$$

Here R_{eff} is the radius of the sphere equal on volume to a rectangular parallelepiped $2L$x$3L$x$5L$, which size L is defined by the mass on entry. Last three columns in Table 2 show: h_l is height where the size L is equal to length of free run of atmospheric molecules, h_{sw} is height, below which the air flow around equivalent sphere of radius R_{eff} occurs in a so-called mode of a thin viscous shock layer, i.e. height, on which for the first time (with reduction of height after the free molecular flow) the thin shock wave is formed. At last, h_{bl} is the height corresponding formation of a thin boundary layer on equivalent sphere. The height h_l was calculated under the following formula

$$h_l = h_0 \ln(l / l_0), \text{ km} \tag{12}$$

Here $l_0 = 0.19 \cdot 10^{-4}$ cm is length of atmospheric molecular free run on the sea level (Цянь Сюэ-сень, 1965). Including $l = L$ with use of the first formula (11), we shall receive

$$h_l = 5.5 \log M_e + 66.7, \text{ km} \tag{13}$$

Value of heights h_{sw} and h_{bl} were calculated on the estimated data resulted in the monograph (Stulov *et al.*, 1995, Fig. B.1 and Fig. 2.1). Approximation of the estimations resulted there gives following formulas

$$h_{sw} = 53 + 17.05 \log R_{eff}, \quad h_{bl} = 40.7 + 15 \log R_{eff}, \text{ km} \tag{14}$$

The data of Table 2 reliably testify that the basic part of luminous sector of the trajectories of investigated fireballs lays in conditions of a flow in a mode of the continuous fluid dynamics, and the condition of free molecular flow ($l \geq L$) is outside of its limits, except fireball 567, h_b = 91.4 km, h_l = 87 km where this condition concerns only to the beginning of the trajectory. In all cases the height of the maximal luminescence is less than height of formation of a powerful head shock wave. At this height reduction of fireball velocity does not exceed 7% from entry velocity. It allows to assume that in these conditions the basic contribution to fireball luminosity gives air in the compressed shock layer. The vapor luminescence has secondary role, so, the known photometric formula (Lebedinets, 1980) is inapplicable.

V. About Dynamic Mass in the Bottom Part of Trajectories

In some cases, observers calculate dynamic mass of a meteoroid in the bottom part of the trajectory with the purpose to receive estimations of possible meteoritics mass (Ceplecha, 1978; Halliday *et al.*, 1996). These calculations are carried out on the basis of braking (negative acceleration) the body received by numerical differentiation of observable dependence $V(t)$. They are not connected in any way with concept of photometric mass.

It is useful to compare these calculations to the values depending on received here initial masses of fireballs from the Canadian network (see Table 1). We shall deal with the analytical device of the exact and approached solutions of the meteoric physics equations described above.

Let's define in the beginning value of braking along a trajectory. Simple calculations with use (3) give

$$\frac{dV}{dt} = \frac{V_e^2 \sin\gamma}{h_0} \frac{v^2 \ln v}{1 + 0.83\beta\, v \ln v} \tag{15}$$

Numerical values of braking (15) and comparison with calculations of observers (Halliday *et al.*, 1996, Table 6) are resulted in Table 3. In the same place, results of numerical differentiation of observational data $V_i\,(t_i)$ using the central differences for internal points are given for the control

$$\frac{dV_i}{dt} = \frac{V_{i+1} - V_{i-1}}{t_{i+1} - t_{i-1}} \tag{16}$$

In table 3, local value of $\sin\gamma$, defined under the formula (10), was used, and the derivative was calculated also by the central difference (16). The sign minus at values of braking is lowered. Data in Tables 3 show that the approximate formula (3) also allows to calculate change of velocity in time with a good accuracy, and it can be used for calculations of braking.

In work of Halliday *et al*. (1996), fireball mass in the bottom part of its trajectory was defined by numerical differentiation of observable distribution of velocity on time in the assumption of the constant body form (a brick-like shape 2*L*x3*L*x5*L*). The solutions of the meteoric physics equations (1) - (3) presented here allow to express change of mass depending on velocity both as the exact solution (the first formula (1)), and through braking (15), and at various assumptions of ablation type. Simple calculations give the following formula

$$m^{1-\mu} = (1 + 0.83\beta\, v \ln v)\exp[-0.83\beta(1-v)] \qquad (17)$$

Here, as well as in the first formula (1), we have: $\mu = 0$ is the movement with the constant middle section area $S = S_e = \mathrm{const}$, $\mu = 2/3$ is the movement with constant factor of body shape $A = A_e = \mathrm{const}$.

So, comparison of dynamic masses in the bottom part of trajectories m_d (the right column in Table 6 of Halliday *et al*., 1996) is carried out with results of calculations under the first formula (1) at $\mu = 2/3$. In some cases, the formula (17) is used also at the same value of μ. Comparison of data is shown on Fig.3 and in Table 4. Continuous lines in Fig. 3 show masses

$$M = M_e f(v,\beta) = M_e \exp[-3\beta(1-v^2)] \qquad (18)$$

calculated under the first formula (1), and M_e are taken from Table 1. Drawing lines show masses M = MI $f(v,\beta)$, where MI are taken from Halliday *et al*., (1996), Table 4. The prevailing part of values MI is made with photometric mass m_p. Stroke-dashed lines on Fig.3 show calculations under the formula (17), where $M = M_e\, m$, kg. In all cases, the dependences $M(v)$ are constructed at $v \geq v_t$, where v_t is a fireball velocity in the last observational point. Small circles on Fig. 3 show values m_d.

In Table 4, values M_1 and M_2 in all cases correspond continuous and drawing lines in Fig. 3, accordingly.

First of all, let's note "attraction" of values m_d to continuous lines, instead of to drawing lines in all cases, except for fireballs 189 and 888 (Table 4) and болида 672 in the same place. In case of fireballs 189 and 888 this "attraction" is not so brightly expressed, as in most cases, and for fireball 672 is absent absolutely. It does not change the general conclusion. The result of comparison once again indirectly refutes conformity of values MI to preatmospheric masses of meteoric bodies. In other words, even rather approximate approach demanding numerical differentiation of observable dependences $V_i(t_i)$ at calculation m_d shows that integration of luminosity, i.e. calculation m_p, cannot give correct values of masses on entry into the atmosphere.

Values of dynamic masses in the bottom part of trajectories are given also in work of Ceplecha (1978). Unfortunately, the method of calculation is described in this work is rather compressed, and trajectories are not resulted. The dynamic mass in the bottom point is calculated under the formula

$$M_2 = \left(\frac{1.2\rho_2 V_2^2}{|\dot{V}|\rho_m^{2/3}} \right)^3 \qquad (19)$$

The structure of the formula shows that M_2 is calculated in the assumption of constant factor of the body form along all trajectory. It is considered that the meteoric body has the form of sphere, and drag coefficient c_d is equal 2. Attention is attracted by a huge difference of values of M_∞ and M_2 (in designations of Tables in work of Ceplecha, (1978)), where M_∞ is calculated under the photometric formula with use of factor τ, which was applied earlier at calculation of photometric masses of fireballs from the Prairie network, USA (McCrosky *et al*., 1979).

It is simple to define the values β providing such significant mass loss. Accepting in (18) $M_e = M_\infty$, $M = M_2$, $v = V_2/V_\infty$, we shall receive values β under Tables in work of Ceplecha, (1978). Results of processing of 17 variants are resulted in Table 5.

It follows from data in Table 5 that condition $M_\infty = M_{ph}$ sharply increases value β. From here it follows that the given condition mismatches the validity, and the parameter β should be defined from approximation of trajectories, as it is made in the present work for fireballs from the Canadian network and in the work (Kulakov and Stulov, 1991) for fireballs from the Prairie network, USA. We shall pay attention to that circumstance that the first line in Table 5 is received at the limiting assumption of full rotation of bodies in flight, when μ = 2/3. At refusal of this assumption, i. e. at μ <2/3 the difference of 1-st line from 2-nd and 3-rd will be stronger.

VI. Gistogramms for Ablation Coefficient from the Prairie and Canadian Networks

Data in Table 1 allow to receive easily values of ablation coefficient $\sigma = 2\beta/V^2$. It is interesting to compare gistogramms of σ for fireballs from the Canadian and Prairie networks. Values of σ for fireballs from the Prairie network were calculated earlier and contain in monograph of Stulov *et al*., (1995). Other variant of a method of the least squares was applied for determination of parameters α and β. The method provides the minimal value of the square-law sum

$$Q_2(\alpha,\beta) = \sum_{i=1}^{n}\left[v(y_i,\alpha,\beta) - v_i\right]^2, \tag{20}$$

where y_i, v_i are observational data, and values of $v(y_i, \alpha, \beta)$ are calculated by means of inverse function of (3). By preparation of this paper, calculations have been lead again by means of a variant of method Q_3, the formula (4) - (8). Values of σ have changed a little, however only in three cases this change exceeds 10%. Results of these calculations σ together with numbers for fireballs from the Prairie network and entry velocity V_e are resulted in Table 6 (McCrosky *et al*., 1979).

Authors of observations of fireballs from the Prairie network specify (McCrosky *et al*., 1978) that in Tables used by us (McCrosky *et al*., 1979), sporadic meteors are basically included, and components of meteoric streams were not considered.

Distribution of fireballs on intervals of σ is resulted in Table 7. The line of the Canadian network contains all fireballs of Tables 1, except for 204 because of extreme value of σ = 0.037 s^2/km^2, i.e. the general number $n = 21$. Lines of the Prairie network contain data for 17 fireballs resulted in Table 6.

The basic conclusion of this comparison consists in the fact that σ for fireballs from the Canadian network are essentially less, than σ for fireballs from the Prairie network. We shall remind communication σ with other, more elementary parameters describing high-speed movement of a body in the atmosphere: $\sigma = c_h/c_d H^*$. The heat exchange coefficient c_h increases with increase in velocity (Stulov *et al*., 1995). Comparison of data in Tables 1 and 6 shows that fireballs from the Canadian network have on the average higher entry velocity, than fireballs from the Prairie network. So, the conditions $V_e < 19$ km/s and $V_e > 19$ km/s give for the Canadian network 11 and 10 fireballs whereas the same quantity for the Prairie network are 14 and 3, accordingly. Therefore values c_h are "on the average" above for fireballs from the Canadian network. As to values of drag coefficient c_d, it is necessary to consider it constant and close to unit if there are no additional independent data about a form of meteoric bodies and their fragments. Therefore the distribution of σ resulted in table 7 testifies to that values of effective enthalpy of evaporation for fireballs from the Canadian network is essential more than this parameter for fireballs from the Prairie network. Apparently, sporadic meteors are on the average less heat-resistant, than components of meteoric streams. Certainly, this important conclusion requires additional check.

VII. Conclusion

Processing of fireballs from the Canadian network has shown that the so-called photometric mass mismatches the physical maintenance of movement of a meteoric body in the atmosphere. Earlier similar results have been received for fireballs from the Prairie network, USA and the Beneshov bolide from the European network (Barry and Stulov, 2003).

Researches of fireballs from the Canadian network will be continued.

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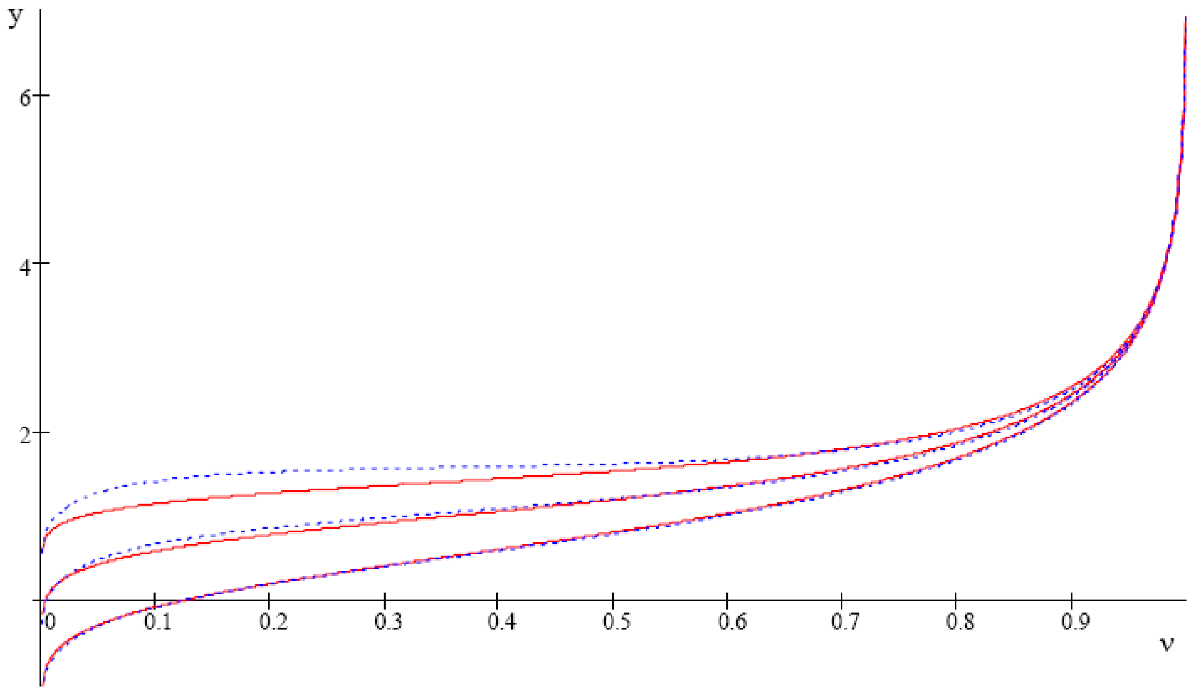

Fig. 1. Approximation of a trajectory by function $Y(\nu)$; continuous lines are the exact solution (1), drawing lines are function $Y(\nu)$ (3).

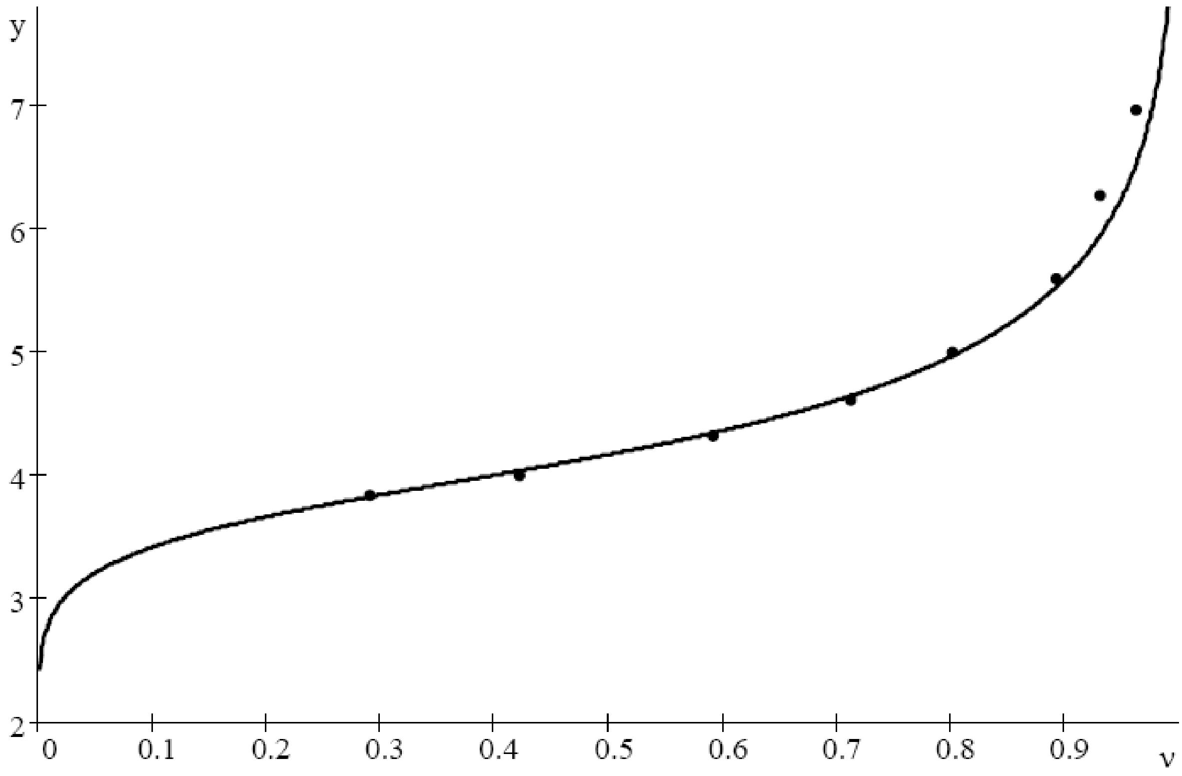

Fig. 2 *a*.

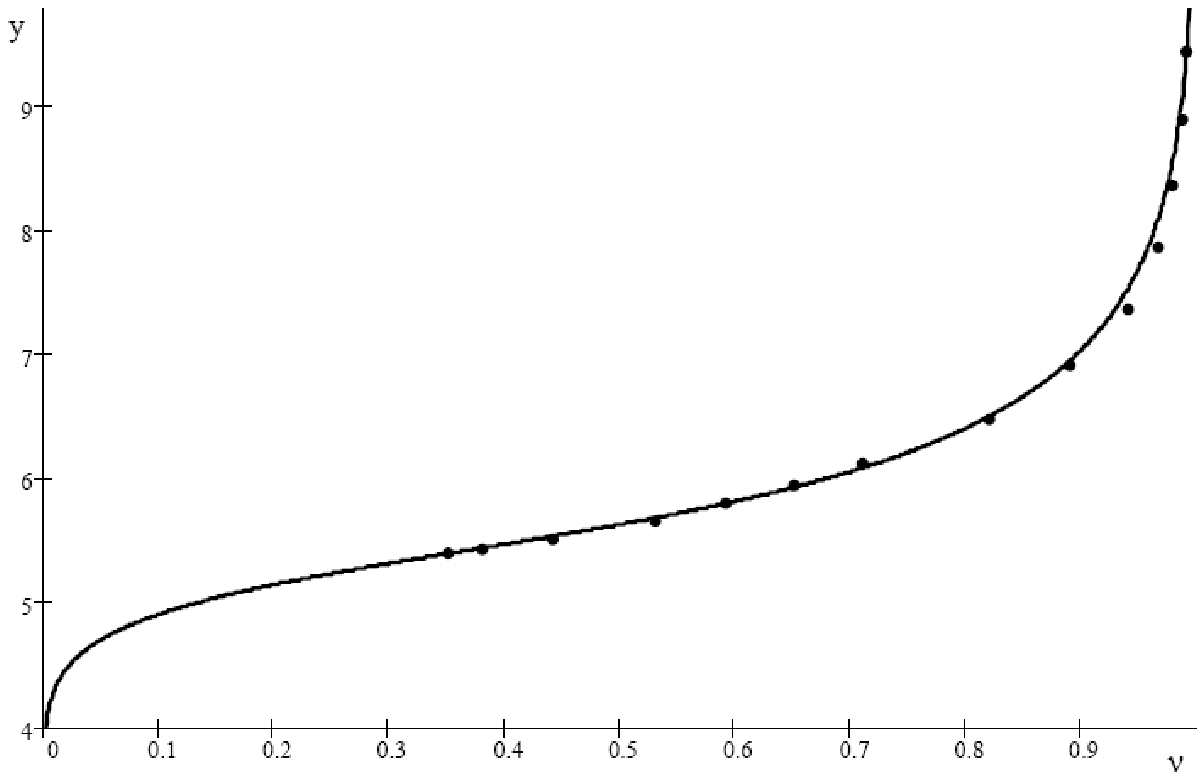

Fig. 2 *b*.

Fig. 2. Approximation of observant trajectories for fireballs 018 (a) and 567 (b); continuous lines are the formula (3), points are data of observation.

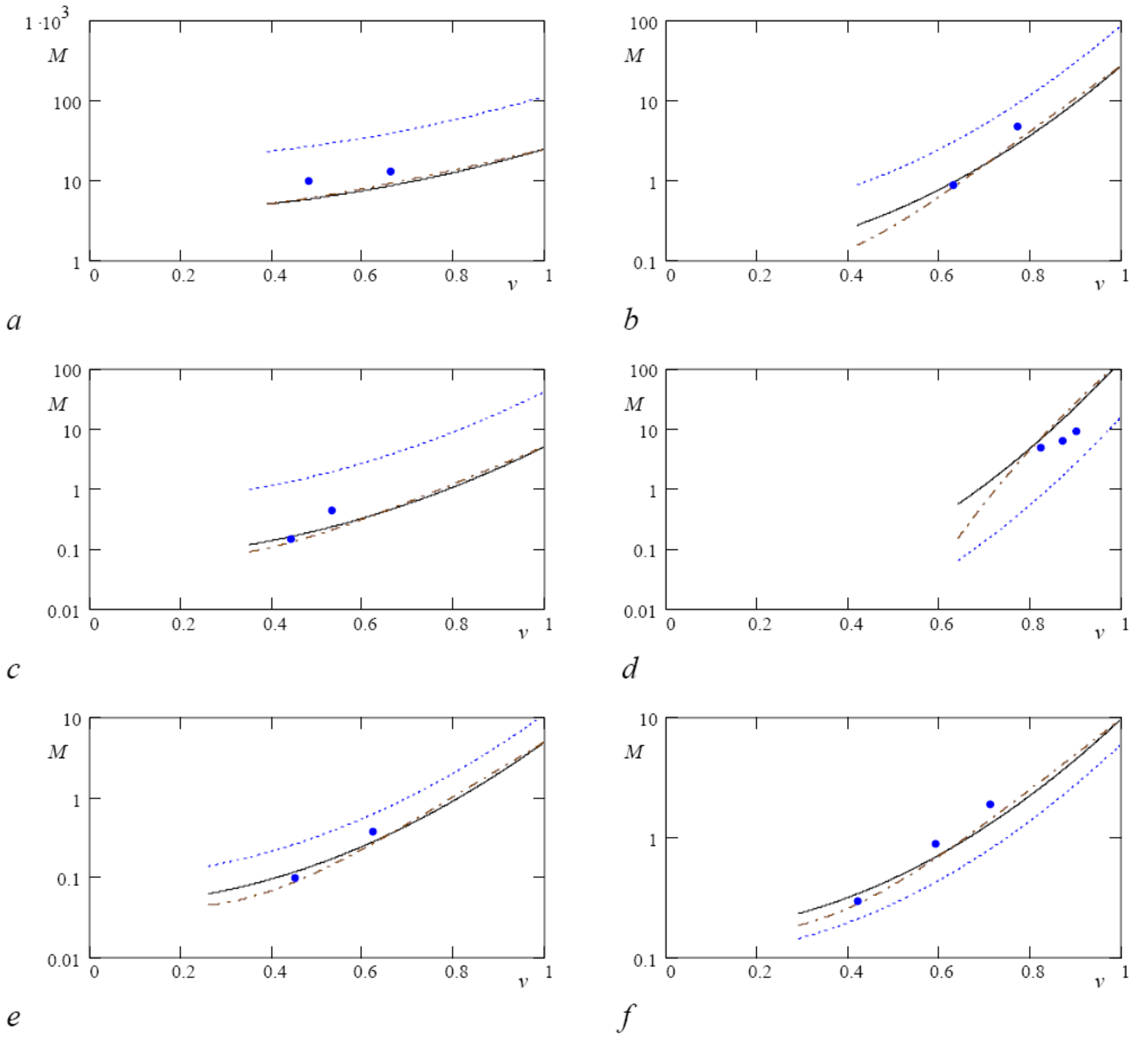

Fig. 3. Dynamic masses for fireballs 872 (a), 219 (b), 567 (c), 204 (d), 840 (e), 018 (f) in the bottom part of their a trajectories.

Table 1. Preatmospheric masses of fireballs from the Canadian network.

№	№ of fireball	V_e, km/s	sinγ	α	β	M_e, kg	MI, kg	m_p, kg
1	925	26.4	0.099	45.11	1.387	338.6	1 300	1 230
2	223	27.1	0.326	19.98	1.596	109.4	240	232
3	872	14.8	0.85	12.59	0.625	24.58	112	80
4	219	18.4	0.772	13.4	1.86	27.28	87	87
5	285	14.5	0.876	8.2	1.746	81.54	51	46
6	567	23.4	0.169	107.15	1.434	5.078	42	42
7	288	12.4	0.716	9.48	1.144	96.65	30	13
8	364	11.3	0.848	20.64	0.575	5.628	21	18
9	204	13.0	0.537	11.18	3.127	139.74	16	2.9
10	169	22.9	0.337	53.45	1.453	5.168	12	12
11	840	23.6	0.754	24.22	1.567	4.95	11	11
12	331	13.3	0.483	38.41	0.615	4.713	10	7.6
13	683	17.6	0.585	38.66	1.314	2.61	10	9.6
14	189	14.5	0.388	35.31	0.741	11.774	9.1	8.1
15	672	13.7	0.591	23.09	1.31	11.894	7.6	4.9
16	195	25.2	0.593	37.17	1.375	2.814	7.1	7
17	205	19.7	1	37.6	0.709	0.548	6.5	6.3
18	018	18.5	0.572	25.5	1.361	9.726	6	5.6
19	307	21.0	0.662	13.05	1.566	46.901	5.7	3.5
20	276	23.5	0.849	19.06	1.153	7.115	5.6	4.8
21	687	16.7	0.339	43.55	0.532	9.411	4.8	1.6
22	888	25.5	0.61	33.27	1.105	3.615	4.6	4.3

Table 2. Characteristic values of height for fireballs trajectories

№ of fireball	V_e, km/s	h_b, km	h_{mI}, km	h_t, km	M_e, kg	L, cm	h_l, km	R_{eff}, cm	h_{sw}, km	h_{bl}, km
018	18.5	75.5	44.9	27.5	9.726	4.5	88.6	8.7	69.0	54.8
223	27.1	78.5	49.0	27.1	109.140	10.1	94.4	19.5	75.0	60.1
567	23.4	91.4	60.0	38.7	5.082	3.6	87.1	6.9	67.3	53.3
925	26.4	91.2	47.6	29.8	338.820	14.7	97.1	28.5	77.8	62.5