Hovering Control of a Solar Sail Gravity Tractor Spacecraft for Asteroid Deflection*

Bong Wie†
Arizona State University, Tempe, AZ 85287

A solar sail gravity tractor (SSGT) spacecraft is proposed as a viable option for deflecting a certain class of near-Earth asteroids (NEAs) such as highly porous rubble piles rather than solid monolithic bodies. Solar sails are large, lightweight reflectors in space that are pushed by sunlight. The SSGT spacecraft concept is based on the so-called Gravity Tractor (GT) for towing asteroids by using gravity as a towline, which was proposed by Lu and Love in the 10 November 2005 issue of Nature. It exploits the "propellantless" nature of solar sails; consequently, its probable advantage over a GT spacecraft propelled by ion engines is its longer mission life times ($>$ 10 years) with a larger "propellantless" ΔV capability. Furthermore, it has no concern of rocket plume impingement on the asteroid surface. This paper demonstrates the practical hovering control feasibility of an SSGT spacecraft for towing NEAs. For example, a 5-year towing of asteroid Apophis using a 2500-kg SSGT spacecraft (equipped with a modest 90-m, 50-kg solar sail of a 0.03-N solar thrust with a 35-deg sun angle) and an additional 3-year coasting time will result in an orbital deflection of 30 km in 2029. A 30-km deflection is more than sufficient to move Apophis out of its 600-m keyhole.

I. Introduction

Early detection, accurate tracking, reliable precision orbit calculation, and characterization of physical properties of near-Earth asteroids (NEAs) are prerequisites to any mitigation mission of deflecting NEAs. Various concepts and approaches for advanced ground-based as well as space-based detection systems are being developed to allow for adequate warning time. Assuming that NEAs on a collision course can be detected prior to impact with a mission lead time of at least 10 years, however, the challenge becomes eliminating their threat, either by fragmenting/destroying the asteroid, or by altering its trajectory so that it will miss Earth. A variety of schemes, including a nuclear standoff detonation, mass drivers, kinetic-energy projectiles, laser beaming, and low-thrust deflection via electric propulsion or solar sails, have been already extensively investigated in the past for such a technically challenging, asteroid mitigation problem.[1] The feasibility of each approach to deflect an incoming hazardous object depends on its size, spin rate, composition, and the mission lead time. Until recently, it was assumed that destruction with thermonuclear weapons would be the most straightforward option in the short term. The recent results from the Hayabusa mission suggest that many asteroids are essentially "rubble piles," rather than solid monolithic bodies. Some experiments show that a thermonuclear detonation within or near such a body of rubble piles would not effectively disperse the (now radioactive) constituent fragments, which would continue following the same trajectory toward Earth.

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†Professor, Dept. of Mechanical & Aerospace Engineering, bong.wie@asu.edu, (480) 965-8674, Fax (480) 965-1384.

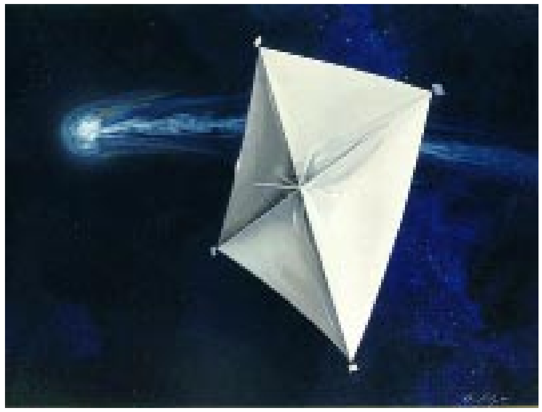

Figure 1. A large solar sail proposed by J. Wright at JPL in 1977 for a rendezvous mission with Halley's comet in a retrograde orbit for the 1986 passage.

Another option would be an impulsive change to the trajectory of the NEA, accomplished either in a single event, or gradually over an extended period. Applied correctly without causing fragmentation of a large asteroid into smaller pieces, the effect of such a ΔV would magnify over decades (or even centuries), eliminating the risk of collision with Earth. A gradual impulsive change might be accomplished by taking advantage of the Yarkovsky effect, in which a rotating asteroid experiences a minute non-radial thrust due to the absorption of sunlight and subsequent re-emission of heat. By varying the reflective and thermal characteristics of one area of an asteroid's surface, thrust could be created in the desired direction. Unfortunately, the requisite technologies for such an operation will not be readily available in the near future. Many of the previously proposed mitigation schemes utilizing such a low-thrust push/pull idea appear to be impractical. These include: attaching large solar sails, mass drivers, or high-efficiency electric propulsion systems to a tumbling or spinning asteroid,[2,3] painting an asteroid to change its albedo to utilize the Yarkovsky effect, and laser beaming to ablate small amounts of material from the surface of a tumbling asteroid. Some of these schemes may also require an extremely large number of a heavy launch vehicle.

A technology does currently exist for a sudden impulsive change, caused by the targeted kinetic impact of a spacecraft on the asteroid's surface. Again, the immediate effect would be small; but if applied long enough prior to a projected Earth impact, the deflection could be sufficient to cause a miss. Thus, the simplest deflection approach is to impact the target NEA with a massive projectile at a high relative speed. However, a successful asteroid deflection mission will require accurate modeling and prediction of the change in velocity caused by such impact. The effective impulse imparted to the asteroid will be the sum of the pure kinetic impulse (linear momentum) of the impactor, plus the impulse due to the thrust of material being ejected from the impact crater. This last term can be very significant (even dominant), but its magnitude depends strongly upon the density and yield strength of the material of which the asteroid is composed, as well as the mass and relative velocity of the impactor. To be most effective, the impacting spacecraft would either have to be massive, or be moving very fast relative to the asteroid. Since current launch technology limits the mass (including propellant) that can be lifted into an interplanetary trajectory, we are therefore led to consider designs that would maximize impact velocity, and which would not require large amounts of fuel.

Propellantless solar sail propulsion, therefore, emerges as a realistic near-term option to such a technically challenging problem of mitigating the threat of NEAs. Solar sails are large, lightweight reflectors in space that are pushed by sunlight.[4,5] A previously proposed concept of using solar sails to tow or tug an asteroid requires an unrealistically large solar sail, which is not technically feasible to assemble in space. Furthermore, attaching such an extremely large solar sail to a tumbling asteroid will not be a simple task. However, solar sails have the potential to provide cost effective, propellantless propulsion that enables longer mission lifetimes, increased payload mass fraction, and access to previously inaccessible orbits (e.g., high solar latitude, retrograde heliocentric, and non-Keplerian). In the past, various solar sailing rendezvous missions with a comet or an asteroid, as illustrated in Fig. 1, have been studied. As illustrated in Fig. 1, a solar sailing concept was studied by JPL in 1977 for a rendezvous mission with Halley's comet for the 1986 passage.[4] Although it soon became an ill-fated mission concept of 1970s, that required a very large, 800-m solar sail to be deployed in space, it introduced the propellantless solar sailing concept to achieve a large, 145-deg orbital inclination change at 0.25 AU in order to rendezvous with Halley's comet in a retrograde orbit. The recent advances in lightweight deployable booms, ultra-lightweight sail films, and small satellite technologies are spurring a renewed interest in solar sailing and the missions it enables.

A solar sailing mission described in Refs. 6-10 utilizes the solar sailing technology to deliver a kinetic energy impactor (KEI) into a heliocentric retrograde orbit, which will result in a head-on collision with a target asteroid at its perihelion, thus increasing its impact velocity to at least 70 km/s. A solar sailing KEI mission architecture, which employs 160-m, 300-kg solar sail spacecraft with a characteristic acceleration of 0.5 mm/s^2, was examined as a realistic near-term option for mitigating the threat posed by NEAs in Refs. 7-10, as illustrated in Fig. 2. For example, a head-on impact (at a relative velocity of 70 km/sec) of a 150 kg impactor on a 200-m, S-class asteroid (with a density of 2,720 kg/m^3) results in a pure kinetic-impact ΔV of approximately 0.1 cm/s. If the asteroid is composed of hard rock, then the modeling of crater ejecta impulse from previous studies would predict an additional ΔV of 0.2 cm/s, which is double the pure kinetic-impact ΔV. However, if the asteroid were composed of soft rock, the previous studies would instead predict an additional ΔV of 0.55 cm/s, which is more than five times the pure kinetic-impact ΔV. Thus, an accurate modeling and prediction of ejecta impulse for various asteroid compositions is a critical part of the most kinetic-impact approaches. For a solar sail KEI mission, its solar sail will be deployed at the beginning of an interplanetary solar sailing phase toward a target asteroid and the KEI spacecraft will be separated from the solar sail prior to impacting a target asteroid. The critical, enabling technologies required for the proposed solar sailing KEI mission include: deployment and control of a 160-m solar sail, development of microspacecraft bus able to withstand the space environment only 0.25 AU from the sun, precision solar sailing navigation, terminal guidance and targeting (accuracy better than 50 m at an impactor speed of 70 km/s), and impact-crater ejecta modeling and accurate ΔV prediction. A 160-m solar sail is not currently available, and the deployment and control of such a large solar sail in space will not be a trivial task.

A practical concern of any kinetic-impact approach of mitigating the threat of asteroids is the risk that the impact could result in the fragmentation of the asteroid, which could substantially increase the damage upon Earth impact. The energy required to fragment an asteroid depends critically upon the asteroid's composition and structure. For example, for a 200-m asteroid composed largely of ice, the disruption energy is approximately 3.4×10^{10} J. Since the kinetic energy of a 150-kg impactor at a relative velocity of 70 km/s would be 3.7×10^{11} J, the 200-m ice asteroid would likely fragment. A 200-m asteroid composed largely of silicates would have a disruption energy of approximately 2.3×10^{12} J, about six times larger than the kinetic energy delivered by the impactor; this asteroid would likely stay intact.

Thus, the feasibility of the most kinetic-impact approaches for deflecting an incoming object depend on its size and composition (e.g., solid body, porous rubble pile, etc.), as well as the time available to change its orbit. An accurate determination of the composition of the target asteroid is a critical part of the kinetic-impact approaches, which may require a separate inspection mission. A further study is also needed to optimize impactor size, relative impact velocity, and the total number of impactors as functions of asteroid size and composition, to ensure a deflection attempt does not cause fragmentation.

Lu and Love[11] has recently proposed an asteroid deflection concept utilizing the mutual gravitational force between a hovering spacecraft and a target asteroid as a towline. For an apparently more fuel-efficient way of towing asteroids, McInnes[12] further discussed the use of a displaced, non-Keplerian orbit rather than a static hovering which requires canted thrusters to avoid plume impingement on the NEA surface. Utilizing the same physical principle of gravitationally "anchoring" a spacecraft to the asteroid, we may employ solar sails rather than nuclear-electric propulsion systems to produce the required continuous low-thrust force.

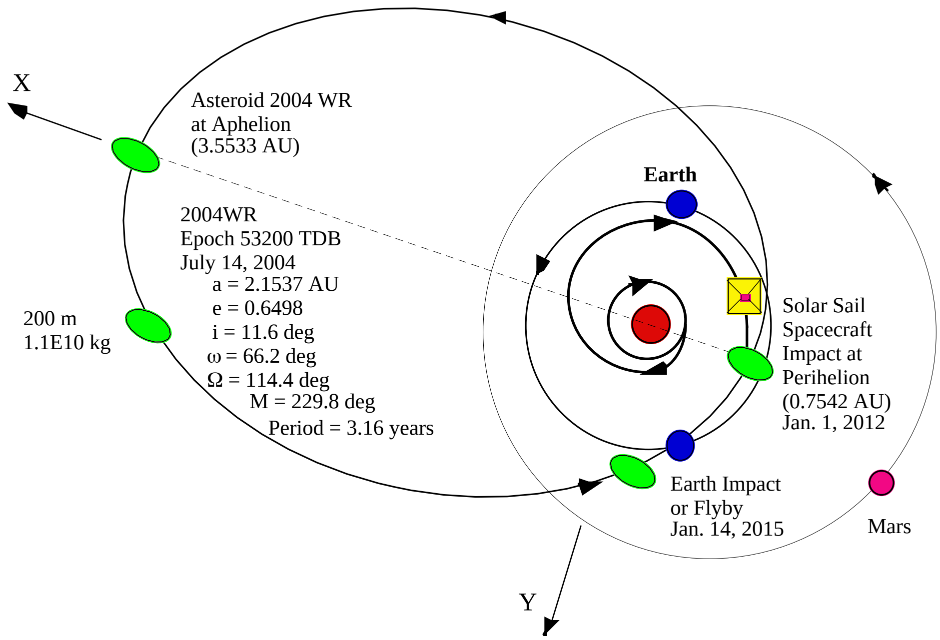

Figure 2. Illustration of the solar sailing KEI mission for impacting and deflecting a near-Earth asteroid. The final, retrograde heliocentric orbit phase (starting from 0.25 AU) results in a head-on collision with the target asteroid at its perihelion of 0.75 AU.[7−9]

The solar sail gravity tractor spacecraft discussed in this paper exploits the "propellantless" nature of solar sails for towing asteroids. The concept of gravitational coupling/towing using solar radiation pressure has been discussed previously for somewhat science-fictional, astronomical problems by Shkadov[13] in 1987 and also by McInnes[14] in 2002.

The remainder of this paper will describe a probable collision threat of asteroid Apophis, the gravity tractor (GT) spacecraft concept, the solar sail gravity tractor (SSGT) options, and the preliminary dynamical modeling and hovering control simulation results. Recent advances in solar sailing technology applicable to the proposed SSCT spacecraft will also be briefly described. In Appendices, asteroid deflection formulas for both the low-thrust towing and kinetic impact approaches will be derived by using the Clohessy-Wiltshire-Hill equations of motion.

II. Asteroid 99942 Apophis

Asteroid Apophis, previously known by its provisional designation 2004 MN4, is a 320-m NEA that is currently predicted to fly by the Earth in 2029 with a possibility of resonant return to impact the Earth in 2036. Apophis is an Aten-class asteroid with an orbital semimajor axis less than 1 AU, and its mass is estimated to be 4.6×10^{10} kg. It has an orbital period of 323 days about the sun. After its close flyby of the Earth in 2029, it will become an Apollo-class asteroid. It was previously predicted that Apophis will pass about 36,350 km from the Earth's surface on April 13, 2029, slightly higher than the 35,786-km altitude of geosynchronous satellites. Recent observations using Doppler radar at the giant Arecibo radio telescope in Puerto Rico have further confirmed that Apophis will in fact swing by at around 32,000 km from the Earth's surface in 2029, but with a very slim chance of resonant return in 2036. A new radar observation at Arecibo Observatory on May 6, 2006 slightly lowered the Palermo scale rating, but the pass in 2036 remained at Torino Scale 1 despite the impact probability dropping to 1 in 24,000. The new orbit estimation increases the 2029 April 13 Earth-center miss-distance by 450 km, from 5.86 (±0.11) to 5.93 (±0.09) Earth radii and reduces the along-track-position uncertainty at closest approach from ±730 to ±570 km (cf. IAUC 8593).

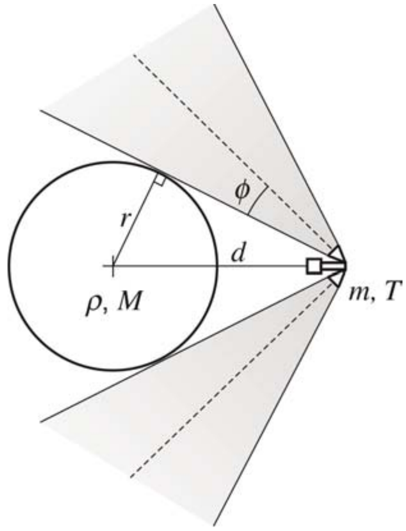

Figure 3. A geometrical illustration of the Gravity Tractor (GT) concept for towing an asteroid.[11]

The nominal predicted Earth-close-approach distance in 2036 increases from 0.168 to 0.276 AU. Further accurate observations of its orbit are expected when it makes fairly close flybys at 0.1 AU from Earth in 2013 and 2021.

Its orbital elements in the J2000 heliocentric ecliptic reference frame are:

$$\begin{aligned}
\text{Epoch} &= \text{JD 2453800.5 TDB (March 6, 2006)} \\
a &= 0.92239 \text{ AU} \\
e &= 0.19104 \\
i &= 3.3312 \text{ deg} \\
\omega &= 126.365 \text{ deg} \\
\Omega &= 204.462 \text{ deg} \\
M &= 222.273 \text{ deg}
\end{aligned}$$

Its other orbital properties are: r_p = 0.746 AU, r_a = 1.0986 AU, v_p = 37.6 km/s, v_a = 25.5 km/s, the orbital period = 323.574 days, the mean orbital rate n = 2.2515e-07 rad/sec, and the mean orbital speed = 30.73 km/s.

As discussed in Refs. 15-16, an extremely small amount of impact ΔV (approximately 0.04 mm/s) in 2026 is required to move Apophis out of a 600-m keyhole area by approximately 10 km in 2029, in case it is going to pass through a keyhole, to completely eliminate any possibility of its resonant return impact with the Earth in 2036. Keyholes are very small regions of the first encounter b-plane such that if an asteroid passes through them, it will have a resonant return impact with the Earth.[17−20] In this paper, asteroid Apophis is used as an illustrative target asteroid assuming that it is going to pass through a 600-m keyhole in 2029.

III. Gravity Tractor for Towing Asteroids

The gravity tractor (GT) concept by Lu and Love[11] utilizes the mutual gravitational force between a hovering spacecraft and a target asteroid as a towline as illustrated in Fig. 3. Although a 20-ton spacecraft propelled by a nuclear-electric propulsion system is considered in Ref. 11, we consider here a 1000-kg spacecraft as an illustrative example applied to asteroid Apophis. To avoid exhaust plume impingement on the asteroid surface, two ion engines are properly tilted outward and the hovering distance is accordingly selected as: $d = 1.5r$ and $\phi = 20$ deg. This illustrative combination yields an engine cant angle of 60 deg, and the two tilted thrusters (each with a thrust T) then produce a total towing thrust T as illustrated in Fig. 3. (For an apparently more fuel-efficient way of towing asteroids, McInnes[12] proposed the use of a displaced, non-Keplerian orbit rather than a static hovering which requires such canted thrusters to avoid plume impingement on the NEA surface.)

A simplified dynamical model for the target asteroid Apophis (ignoring its orbital motion) is

$$M\frac{\Delta V}{\Delta t} = \frac{GMm}{d^2} = T \tag{1}$$

or

$$\frac{\Delta V}{\Delta t} = \frac{Gm}{d^2} = \frac{T}{M} = A \tag{2}$$

where $G = 6.6695 \times 10^{-11}$ N-m^2/kg^2, $M = 4.6 \times 10^{10}$ kg, $m = 1000$ kg, r = 160 m. d = 240 m, $T = 0.05326$ N, $A = 1.1579 \times 10^{-9}$ mm/s^2 is the characteristic acceleration, and

$$\Delta V = A\Delta t \tag{3a}$$

$$\Delta X = \frac{1}{2}A(\Delta t)^2 \tag{3b}$$

where ΔV and ΔX are, respectively, the resulting velocity and position changes for the total towing period of Δt. For example, we have ΔV = 0.0365 mm/s and ΔX = 575 m for Δt = one year.

Including the orbital "amplification" effect (see Appendix A), we have

$$\Delta V = 3A\Delta t \tag{4a}$$

$$\Delta X = \frac{3}{2}A(\Delta t)^2 \tag{4b}$$

Consequently, we have ΔV = 0.1095 mm/s and ΔX = 1.7 km for one-year towing.

Including an additional coasting time of t_c, we have the total position change given by

$$\Delta X = \frac{3}{2}A\Delta t(\Delta t + 2t_c) \tag{5}$$

A derivation of this deflection formula can be found in Appendix A. Thus, one-year towing in 2026 will cause a total position change of approximately 12 km in 2029, which is more than sufficient to move Apophis out of its 600-m keyhole in 2029.

The propellant amount required for maintaining a desired hovering altitude of 80 m can be estimated as

$$\Delta m_f = \frac{2T\Delta t}{g_o I_{sp}} \approx 0.3 \text{ kg per day } \approx 114 \text{ kg per year}$$

where $T = 0.053$ N, $g_o = 9.8$ m/s^2, and $I_{sp} = 3000$ sec (assumed for typical ion engines).

Therefore, a 1000-kg GT spacecraft equipped with ion engines can be considered as a viable option for a pre-2029 deflection mission for Apophis. However, it is emphasized that a 1000-kg spacecraft, colliding with Apophis at a modest impact velocity of 10 km/s in 2026, will cause a much larger, instantaneous velocity change of at least 0.22 mm/s for Apophis, resulting in an orbital deflection of 62 km in 2029. Such a high-energy kinetic impactor approach may not be applicable to highly porous, rubble-pile asteroids and a GT spacecraft mission may need an additional large ΔV to rendezvous with a target asteroid. Consequently, a further study on various issues, such as the total mission ΔV requirement, low-thrust gravity towing vs. high-energy kinetic impact, asteroid dispersal/fragmentation concern, etc., is needed.

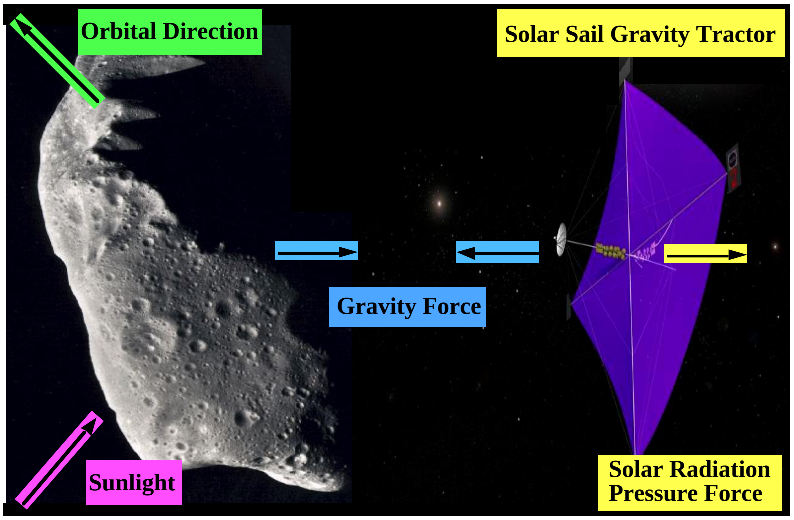

Figure 4. An illustration of the solar sail gravity tractor (SSGT) concept for asteroid deflection.

IV. Solar Sail Gravity Tractor (SSGT) Options

Utilizing the same physical principle of employing the mutual gravitational force between a hovering spacecraft and a target asteroid as a towline,[11] we may choose solar sails rather than nuclear-electric propulsion systems to produce the required continuous low-thrust towing force. The basic physical principle of the SSGT spacecraft hovering over the NEA surface is illustrated in Fig. 4. The concept of gravitational coupling/towing by the use of solar radiation pressure has been proposed previously for somewhat larger scale (science-fictional) astronomical problems by Shkadov[13] in 1987 and also by McInnes[14] in 2002. Solar sails are large, lightweight reflectors in space that are pushed by sunlight. The SSGT spacecraft discussed in this section exploits the "propellantless" nature of solar sails for towing asteroids; consequently its probable advantage over a GT spacecraft propelled by ion engines is its longer mission life times ($>$ 10 years) with a larger "propellantless" ΔV capability. Furthermore, it has no concern of rocket plume impingement on the asteroid surface.

A NEA deflection system architecture consisting of GT spacecraft options, SSGT spacecraft options, a solar sail kinetic energy impactor (KEI), and a NEO orbiting surveyor spacecraft is illustrated in Fig. 5. For a solar sail KEI mission, its solar sail will be deployed at the beginning of an interplanetary solar sailing phase toward a target asteroid and the KEI spacecraft will be separated from the solar sail prior to impacting a target asteroid. For the SSGT spacecraft mission, its solar sail will be deployed after completing a rendezvous with a target asteroid.

As illustrated in Fig. 5, three different options of GT/SSGT spacecraft are possible as follows:

- Baseline: a GT with two canted ion engines, hovering at $(x, y) = (240, 0)$ m
- Option 1: a GT with orthogonally mounted ion engines, hovering at $(x, y) = (200, 200)$ m
- Option 2: an SSGT with an ion engine and a solar sail, hovering at $(x, y) = (200, 200)$ m
- Option 3: an SSGT with a solar sail (35-deg sun angle), hovering at $(x, y) = (286, 409)$ m

As illustrated in Fig. 5, it is feasible to employ a GT with two orthogonally mounted ion engines (Option 1). Because only the x-axis thrust force provides an effective ΔV of the target asteroid, the y-axis ion engine of the Option-1 GT can be replaced by a 70-m solar sail resulting in an Option-2 SSGT spacecraft.

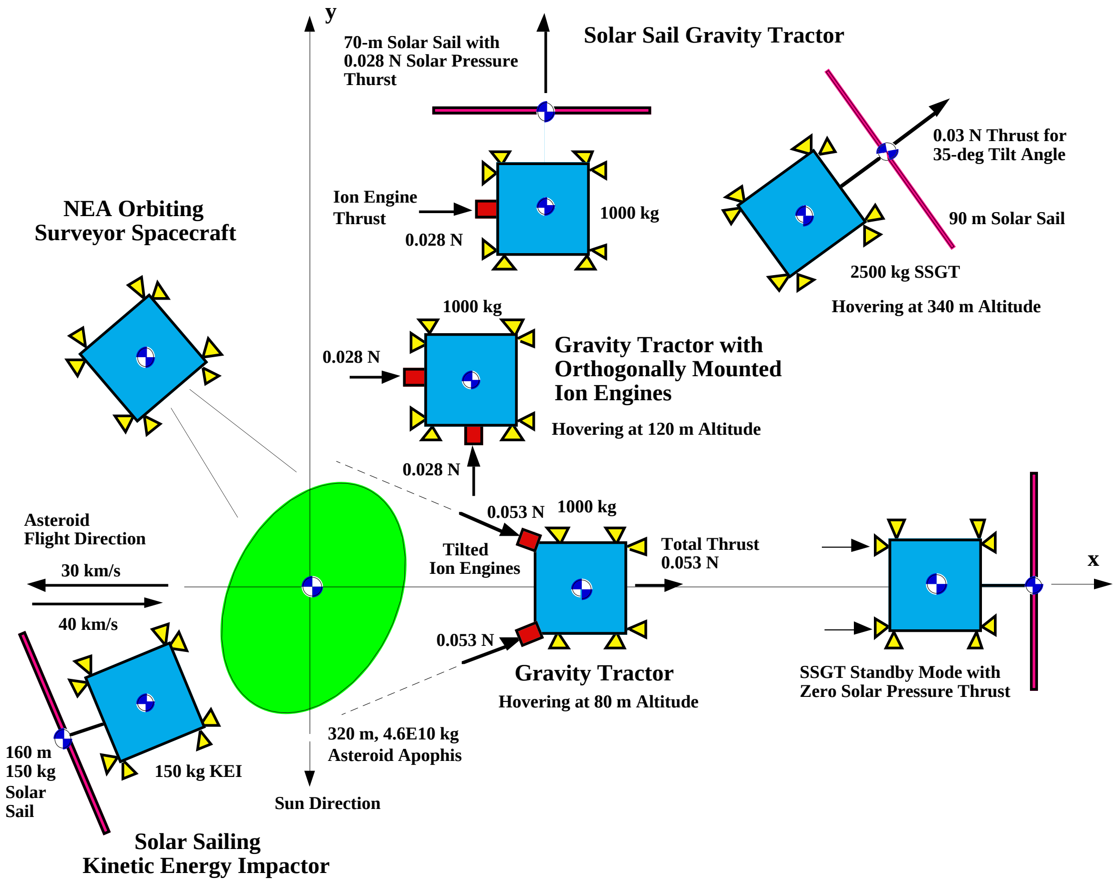

Figure 5. NEA deflection system architecture consisting of a gravity tractor (w/o solar sail), a solar sail gravity tractor (SSGT), a solar sail kinetic energy impactor (KEI), and a NEA orbiting surveyor spacecraft.

An Option-3 SSGT spacecraft propelled using only a solar sail (35-deg sun angle) is also proposed for a case which may require much longer mission lifetimes (> 5 years) with a much larger "propellantless" ΔV capability. However, the Option-3 SSGT spacecraft requires a heavier, 2500-kg spacecraft with a 90-m solar sail to be able to hover at a slightly higher altitude of 340 m, compared to the other spacecraft hovering at an altitude of 120-m. In practice, multiple ion engines and redundant position/attitude control thrusters will be required for the GT/SSGT spacecraft, and thus a detailed system-level tradeoff study (e.g., ion engines vs solar sails) is needed for the GT/SSGT spacecraft design.

As an example, consider a 2500-kg SSGT spacecraft, equipped with a modest 90-m, 50-kg solar sail of a 0.03-N solar thrust with a 35-deg sun angle which produces an along-track acceleration of $A = 3.74 \times 10^{-10}$ mm/s^2. A 5-year towing of Apophis using this 2500-kg SSGT spacecraft and a 3-year coasting time will result in an orbital deflection of 30 km in 2029, which is more than sufficient to move Apophis out of its 600-m keyhole.

In the next section, simplified dynamical modeling and hovering control of the GT/SSGT spacecraft will be presented to validate the technical feasibility of the GT/SSGT concept for towing asteroids.

V. Modeling and Control of Hovering SSGT Spacecraft

A simple planar model of the hovering dynamics of an SSGT spacecraft towing a target asteroid is illustrated in Fig. 6. Utilizing the Clohessy-Wiltshire-Hill equations of motion in astrodynamics,[21] we can derive the equations of motion of the asteroid-spacecraft system orbiting around the sun as follows:

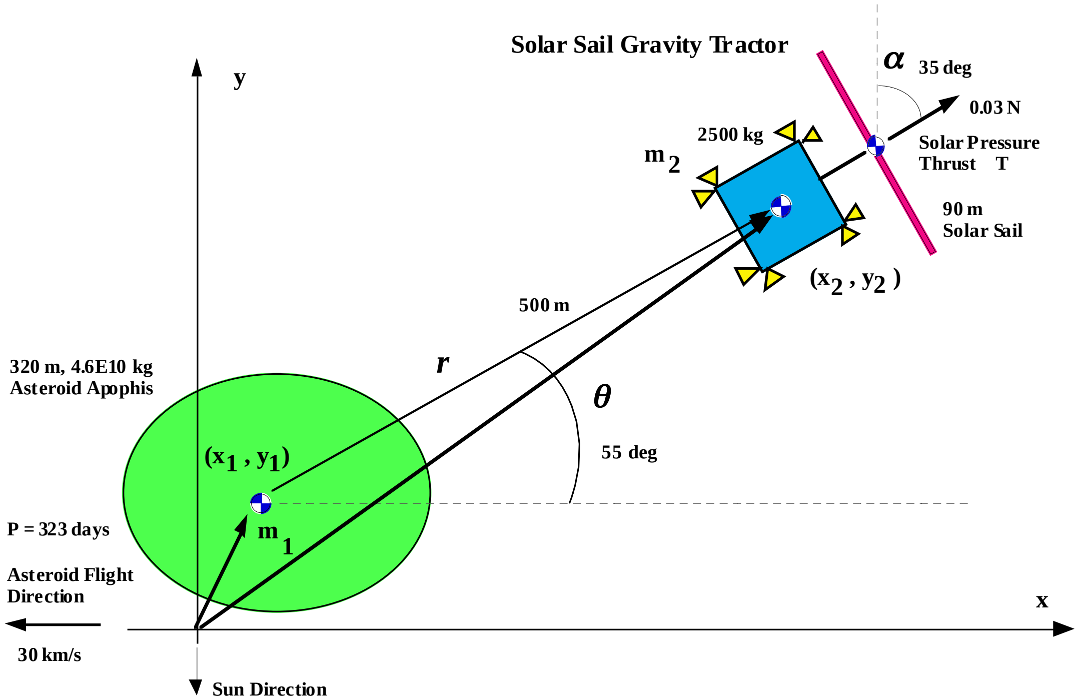

Figure 6. A simplified dynamical model for hovering control analysis of a solar sail gravity tractor (SSGT) spacecraft.

$$\ddot{x}_1 = 2n\dot{y}_1 + Gm_2 \frac{x_2 - x_1}{r^3}(1 + E_x) \tag{6}$$

$$\ddot{y}_1 = -2n\dot{x}_1 + 3n^2 y_1 + Gm_2 \frac{y_2 - y_1}{r^3}(1 + E_y) \tag{7}$$

$$\ddot{x}_2 = 2n\dot{y}_2 - Gm_1 \frac{x_2 - x_1}{r^3}(1 + E_x) + \frac{1}{m_2}(T_x + F_x) \tag{8}$$

$$\ddot{y}_2 = -2n\dot{x}_2 + 3n^2 y_2 - Gm_1 \frac{y_2 - y_1}{r^3}(1 + E_y) + + \frac{1}{m_2}(T_y + F_y) \tag{9}$$

where (x_1, y_1) are the coordinates of the target asteroid with respect to an orbiting reference frame, (x_2, y_2) the coordinates of the SSGT spacecraft, (T_x, T_y) solar pressure thrust components, (F_x, F_y) control thrust components, (E_x, E_y) the gravitational perturbations caused by a spinning motion of an irregularly shaped asteroid, $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2)}$, $G = 6.6695 \times 10^{-11}$ N-m^2/kg^2, m_1 the asteroid mass, m_2 the SSGT spacecraft mass, and n the orbital rate of the reference frame (x, y). For simplicity, a circular orbital motion of the reference frame is considered here. The eccentric orbital effect of a target asteroid will be discussed in Appendix B.

Preliminary hovering control logic is considered as follows:

$$\begin{aligned}
\tan\theta &= \frac{y_2 - y_1}{x_2 - x_1} \\
\alpha &= \pi/2 - \theta \\
T_x &= T_o \cos^2\alpha \sin\alpha \\
T_y &= T_o \cos^2\alpha \cos\alpha \\
x &= x_2 - x_1 = r\cos\theta \\
y &= y_2 - y_1 = r\sin\theta
\end{aligned}$$

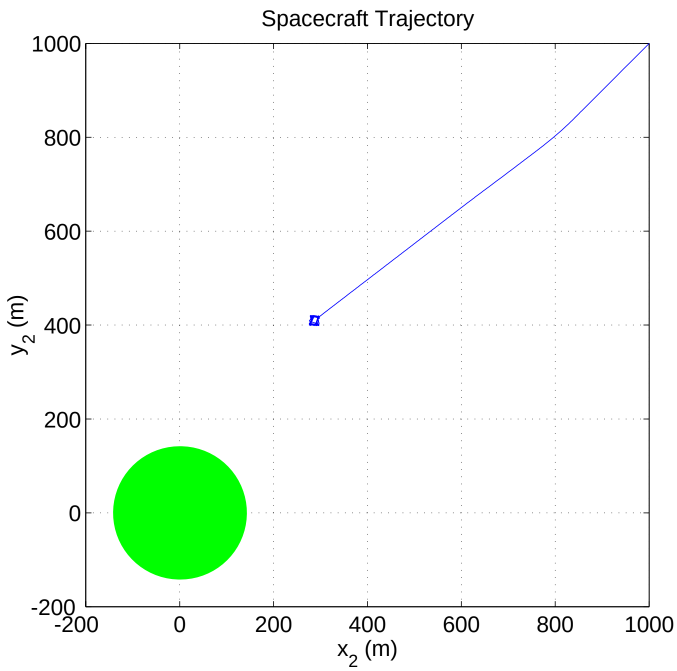

Figure 7. Hovering control simulation results for the SSGT (Option 3). Starting point at (x, y) = (1000, 1000) m and a desired hovering point (x, y) = (286, 409) m.

$$
\begin{aligned}
F_x =& -K_p(x - x_c) - K_d\dot{x} \\
F_y =& -K_p(y - y_c) - K_d\dot{y} \\
& \quad \text{if } |F_x| > F_{max},\ F_x = \text{sgn}(F_x)F_{max} \\
& \quad \text{if } |F_y| > F_{max},\ F_y = \text{sgn}(F_y)F_{max} \\
& \quad \text{if } |x - x_c| < \epsilon_x,\ F_x = 0 \\
& \quad \text{if } |y - y_c| < \epsilon_y,\ F_y = 0
\end{aligned}
$$

where $T_o = 0.045$ N for a 90-m solar sail at 1 AU, $F_{max} = 0.1$ N, $\epsilon_x = \epsilon_y = 10$ m, and (x_c, y_c) is the desired hovering position command, $K_p = 0.00001m_2$, and $K_d = 0.03m_2$, For simplicity, electric thrusters with a maximum thrust of 0.1 N and a specific impulse of 3000 sec are considered for the hovering control of GT/SSGT spacecraft. An attitude control problem of the GT/SSGT spacecraft is not considered in this paper.

Hovering control simulation results are shown in Figs. 7-11 for a 2500-kg SSGT spacecraft (Option 3). For these simulations, $\pm 20\%$ cyclic gravitational perturbations (caused by a spinning motion of an irregularly shaped asteroid) are simply modeled as: $E_x = 0.2 \sin \Omega t$ and $E_y = 0.2 \cos \Omega t$ where Ω is the spin rate of the target asteroid with an assumed spin period of 5 hrs. As can be estimated from Fig. 11, less than 7-kg propellant per year (worst case) may be needed for a 2500-kg SSGT spacecraft to hover above a target asteroid with its quite uncertain gravitational environment. Any undesirable cyclic thruster firings can be easily eliminated by employing a cyclic-disturbance rejection control scheme (Ref. 21) to reduce the hovering control propellant consumption to less than 2 kg per year. However, a more rigorous dynamical model of the gravitational field of a slowly rotating irregularly shaped asteroid must be used for the detailed hovering control design and high-fidelity simulations, as discussed in Refs. 22 and 23. The Yarkovsky effect must also be included in such a rigorous dynamical model.

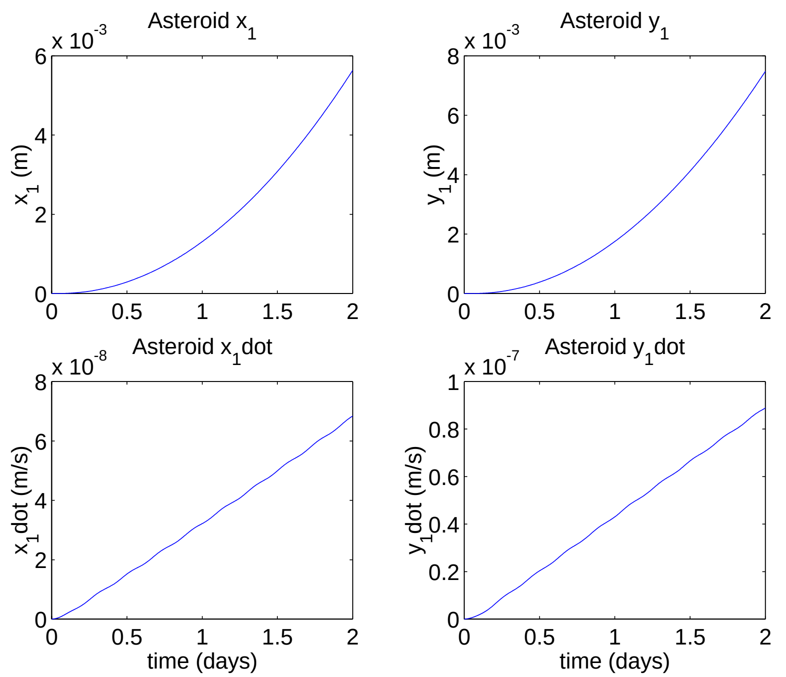

Figure 8. Hovering control simulation results for the SSGT (continued).

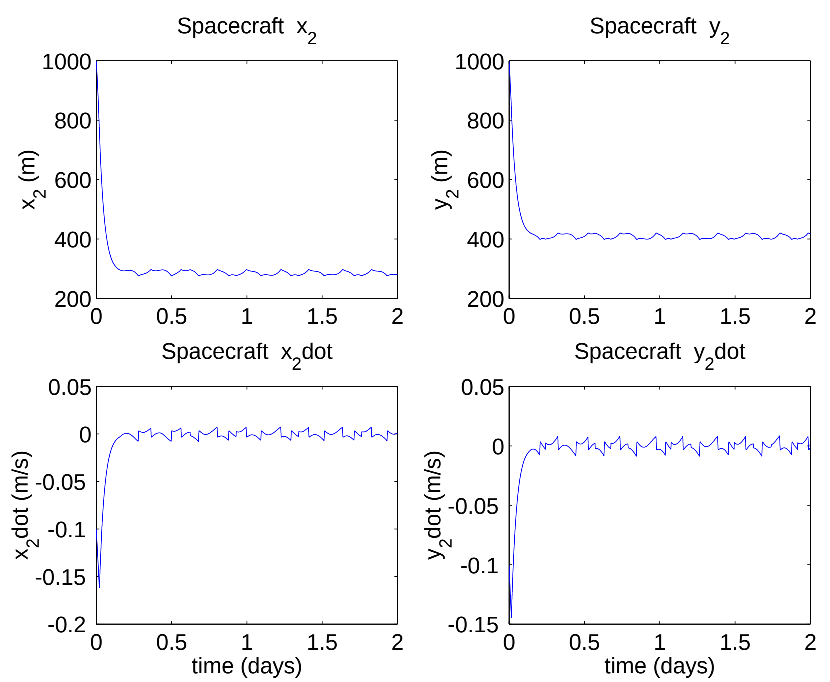

Figure 9. Hovering control simulation results for the SSGT (continued).

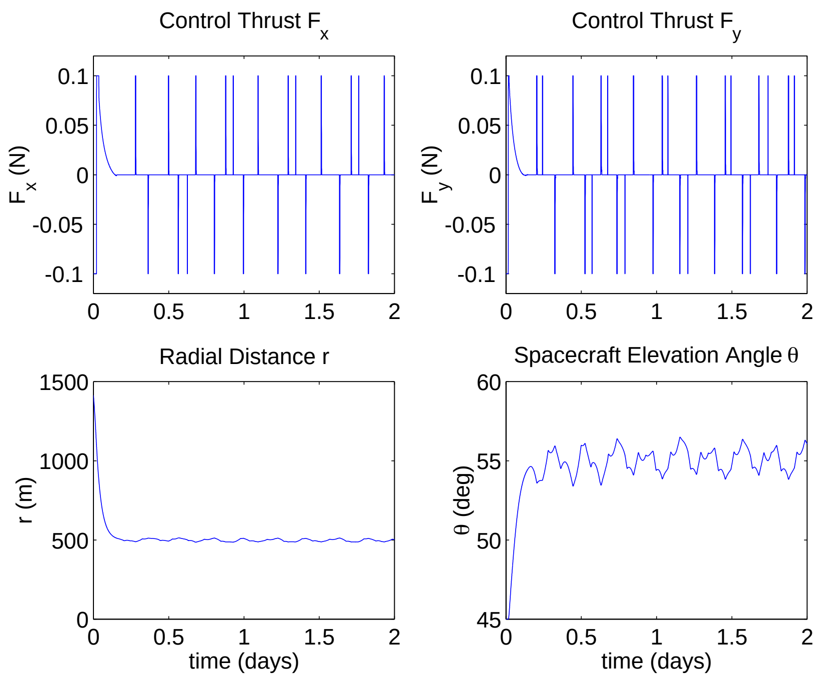

Figure 10. Hovering control simulation results for the SSGT (continued).

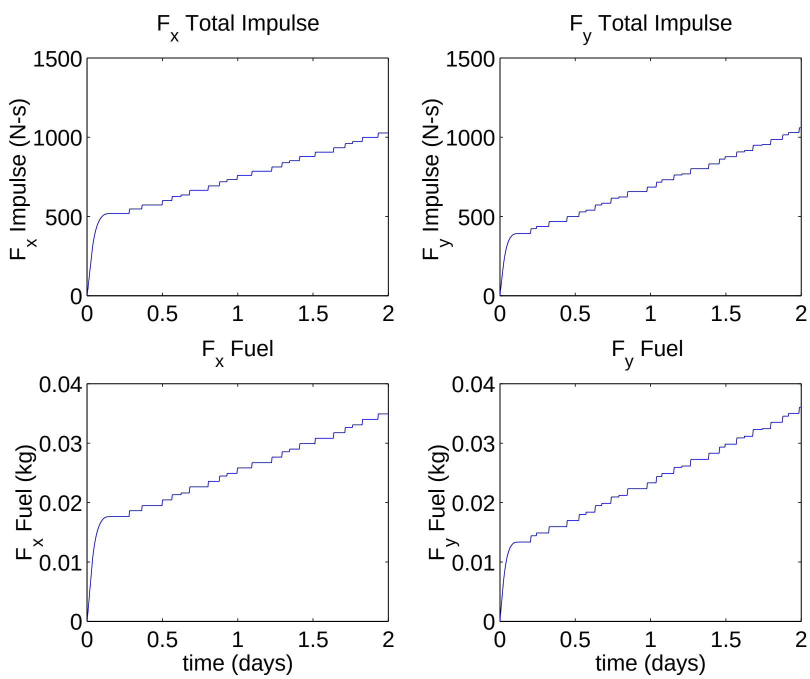

Figure 11. Hovering control simulation results for the SSGT (continued).

VI. Recent Advances in Solar Sailing Technology

The recent advances in lightweight deployable booms, ultra-lightweight sail films, and small satellite technologies are spurring a renewed interest in solar sailing and the missions it enables. Consequently, various near-term solar sailing missions and the associated technologies are being developed.[24–27] A 100-m class solar sail required to propel the SSGT spacecraft is not currently available, and the deployment and control of such a large solar sail in space will not be a trivial task. However, a 20-m solar sail ground validation project[25–26] of NASA's In-Space Propulsion Technology Program has been successfully completed in 2005. A space flight experiment of the 30-m, 105-kg Cosmos 1 solar sail spacecraft was attempted by The Planetary Society on June 21, 2005. Because of a boost rocket failure, the Cosmos 1 solar sail project did not achieve its mission goal of demonstrating the first controlled solar sail flight as the spacecraft is propelled by photons from sunlight. A 40-m solar sail is currently being developed by NASA and industries for a possible flight validation experiment via the New Millennium Program (NMP) Space Technology 9 program within 10 years. A 160-m solar sail will be required for NASA's Solar Polar Imager (SPI) mission, which is one of the Sun-Earth Connections solar sail roadmap missions currently envisioned by NASA, as illustrated in Fig. 12. Consequently, it is expected that a 100-m class solar sail will be available within 15 years.

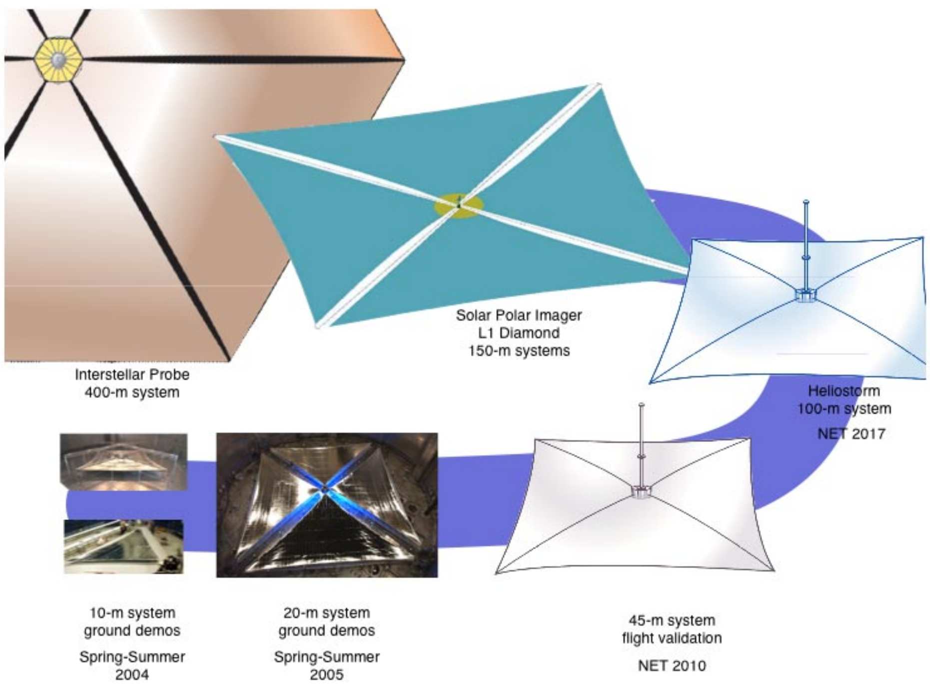

Figure 12. NASA's solar sail roadmap. Image courtesy of NASA.

VII. Conclusions

A solar sail gravity tractor (SSGT) spacecraft has been presented as an option for deflecting a certain class of asteroids such as highly porous, rubble pile asteroids. The SSGT spacecraft concept was based on the gravity tractor (GT) for towing asteroids, proposed by Lu and Love in the 10 November 2005 issue of Nature; it further exploits the "propellantless" character of solar sails. The practical hovering control feasibility of a 2500-kg SSGT spacecraft has been demonstrated for towing Apophis. It is emphasized that a 2500-kg spacecraft with a relative impact velocity of 10 km/s can simply generate a much larger, instantaneous velocity change of at least 0.5 mm/s for Apophis. However, such a kinetic energy impactor approach may not be applicable to highly porous, rubble-pile asteroids. Thus, detailed system-level studies on various issues, such as asteroid fragmentation/dispersal concern, low-thrust gravity towing vs. high-energy kinetic impact, ion engines vs. solar sails, etc., are needed. Throughout this paper, asteroid Apophis was used as an illustrative target asteroid assuming that it is going to pass through a 600-m keyhole in 2029.

Acknowledgments

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Appendix A: Orbital Amplification Effect on the Miss Distance

Consider the Clohessy-Wiltshire-Hill equations of motion:

$$\ddot{x} = 2n\dot{y} + A_x \tag{10}$$

$$\ddot{y} = -2n\dot{x} + 3n^2 y + A_y \tag{11}$$

where (x, y) are the coordinates of an asteroid with respect to a circular orbit reference frame shown in Fig. 6 and (A_x, A_y) are the control acceleration components acting on the asteroid. The out-of-plane motion is not considered here. An ideal case with $A_x = A$ = constant and $A_y = 0$ is further assumed here.

Integrating the x-axis equation, we obtain

$$\dot{x} = \dot{x}(0) + 2ny + At \tag{12}$$

where $\dot{x}(0)$ denotes the along-track velocity at $t = 0^-$. All other initial conditions will be ignored here. For a kinetic energy impactor problem, the initial impact ΔV along the x-axis direction becomes $\dot{x}(0)$.

Substituting Eq. 12 into the y-axis equation, we obtain

$$\ddot{y} + n^2 y = -2n\dot{x}(0) - 2nAt \tag{13}$$

Its solution can be found as

$$y(t) = -\frac{2}{n}\dot{x}(0)(1 - \cos nt) - \frac{2}{n}A\left(t - \frac{1}{n}\sin nt\right) \tag{14}$$

and

$$\dot{y}(t) = -2\dot{x}(0)\sin nt - \frac{2}{n}A(1 - \cos nt) \tag{15}$$

We then obtain

$$\dot{x}(t) = -\dot{x}(0)(3 - 4\cos nt) - 3At + \frac{4}{n}A\sin nt \tag{16}$$

which can be integrated as

$$x(t) = -\dot{x}(0)\left(3t - \frac{4}{n}\sin nt\right) - \frac{3}{2}At^2 + \frac{4}{n^2}A(1 - \cos nt) \tag{17a}$$

$$\approx -3\dot{x}(0)t - \frac{3}{2}At^2 \quad \text{for large t} \tag{17b}$$

The orbital "amplification" factor of three can be seen from the preceding equation. Note that the positive values of $\dot{x}(0)$ and A slow down the asteroid and reduce its orbital energy. Consequently, its along-track position becomes negative (i.e., ahead of its unperturbed virtual position in a circular reference orbit).

Consider an asteroid with the accelerated towing time of t_a and the additional coasting time of t_c. It is assumed that $\dot{x}(0) = 0$ here. A new set of initial conditions at the end of towing period become:

$$\begin{aligned}
x_0 &= -\frac{3}{2}At_a^2 + \frac{4}{n^2}A(1 - \cos nt_a) \\
\dot{x}_0 &= -3At_a + \frac{4}{n}A\sin nt_a \\
y_0 &= -\frac{2}{n}A\left(t_a - \frac{1}{n}\sin nt_a\right) \\
\dot{y}_0 &= -\frac{2}{n}A(1 - \cos nt_a)
\end{aligned}$$

The final position changes at the end of the coasting phase can then be found as

$$\Delta x = x_0 + (6ny_0 - 3\dot{x}_0)t_c + \frac{2\dot{y}_0}{n}(1 - \cos nt_c) + \left(\frac{4\dot{x}_0}{n} - 6y_0\right)\sin nt_c \tag{18}$$

$$\Delta y = 4y_0 - \frac{2\dot{x}_0}{n} + \left(\frac{2\dot{x}_0}{n} - 3y_0\right)\cos nt_c + \frac{\dot{y}_0}{n}\sin nt_c \tag{19}$$

Substituting the initial conditions into Eq. 18, we obtain

$$\Delta x \approx -\frac{3}{2}At_a(t_a + 2t_c) \tag{20}$$

which is the low-thrust deflection formula discussed in Refs. 28-30 using different approaches. Note that $\Delta y \approx 0$ compared to Δx.

Equation 20 can be rewritten as

$$\Delta x = -\left(\frac{3}{2}At_a^2 + \Delta V t_c\right) \quad \text{where } \Delta V = 3At_a \tag{21}$$

Note that Δx is caused by various initial conditions including $\dot{x}_0$ and y_0 as can be seen in Eq. 18. Such a combined effect of $\dot{x}_0$ and y_0 results in the term $\Delta V t_c$ (not $3\Delta V t_c$ as one might expect) in Eq. 21.

For an asteroid in an eccentric orbit colliding with the earth, we have

$$V = V_\oplus\sqrt{2 - \frac{r_\oplus}{a}} \tag{22}$$

$$e^2 = (\lambda - 1)^2\cos^2\gamma + \sin^2\gamma \tag{23}$$

where V is its heliocentric velocity at an impact point, a its semimajor axis, e its eccentricity, $r_\oplus$ = 1 AU = 1.496×10^8 km, $V_\oplus$ = 29.784 km/s, and γ the intersection angle between $\vec{V}$ and $\vec{V}_\oplus$, and $\lambda = (V/V_\oplus)^2$. The heliocentric elevation angle γ is also called the flight path angle.

As illustrated in Fig. 13 for a case with $a \approx r_\oplus$, we have

$$V \approx V_\oplus$$

$$e \approx \sin\gamma$$

$$d = \Delta x\cos(\gamma/2)$$

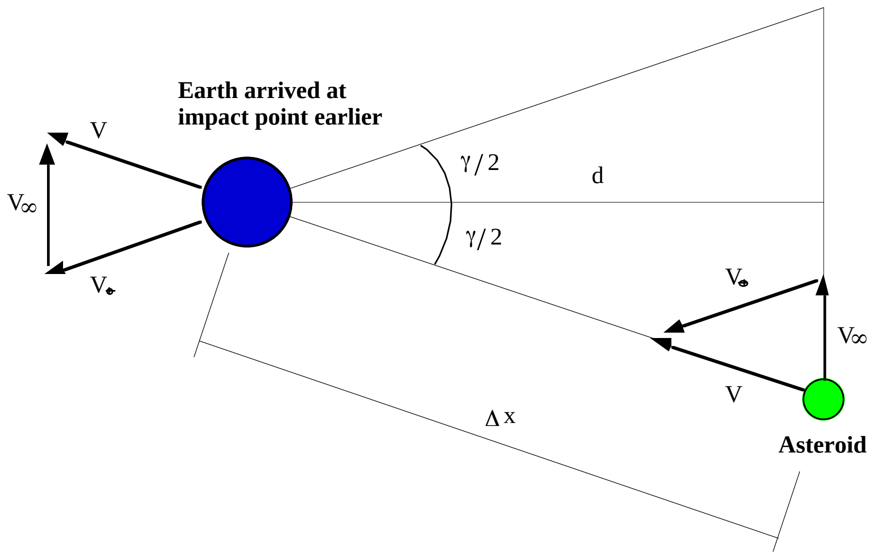

Figure 13. Asteroid deflection geometry for a case with $a \approx 1$ AU.

where d is the approach distance (also called the b-plane miss distance). The impact parameter b, which defines the radius of a collision cross section, is given by

$$b = R_{\oplus}\sqrt{1 + \frac{V_e^2}{V_\infty^2}} \tag{24}$$

where $R_{\oplus}$ is the radius of the earth (= 6378 km/s), $V_e = \sqrt{2\mu_{\oplus}/R_{\oplus}}$ the escape velocity from the surface of the earth (= 11.18 km/s), and V_∞ the hyperbolic approach velocity. To avoid an impact, we simply need $d > b$.

For a somewhat unusual case of a near head-on collision of an asteroid (or comet) with the earth, we have

$$d = \Delta x \sin(\gamma/2) \approx -\frac{3e}{4} A t_a (t_a + 2t_c)$$

Obviously, it will be extremely difficult to deflect a NEO which is in a head-on collision orbital path toward the earth.

For an impulsive ΔV along the x-axis direction, the resulting deflection Δx after a coasting time of t_c is simply given by

$$\Delta x = -3 \Delta V t_c \tag{25}$$

For a kinetic impactor approach, ΔV can be estimated as

$$\Delta V \approx \eta \frac{m}{M + m} U \approx \eta \frac{m}{M} U \tag{26}$$

where η is the impact efficiency factor, m the impactor mass, M the target asteroid mass, and U the relative impact velocity.

Consider a numerical test case (Option 3 SSGT) for an asteroid with a circular orbital period of 323 days (n = 2.2515×10^{-7} rad/sec), an assumed low-thrust acceleration of $A_x = 3.8284 \times 10^{-10}$ mm/s^2, $A_y = 5.4667 \times 10^{-10}$ mm/s^2, t_a = 5 years, and t_c = 3 years. The final position change Δx can be estimated as -30 km, as can also be noticed in Fig. 14.

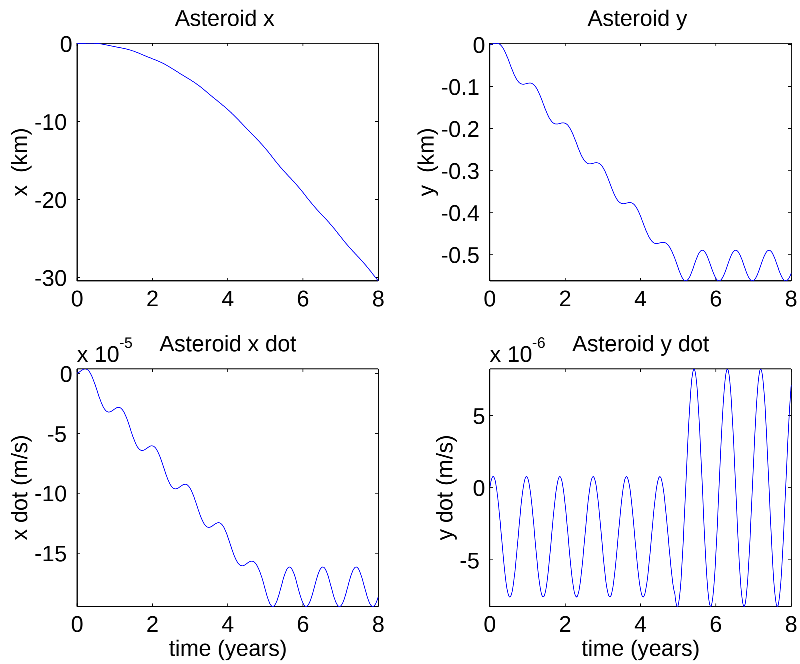

Figure 14. Long-term simulation of the SSGT for Apophis in an assumed circular obit.

Appendix B: Eccentricity Effect on the Miss Distance

The Clohessy-Wiltshire-Hill equations of motion relative to an eccentric reference orbit are given by:

$$\ddot{x} = 2\dot{\theta}\dot{y} + \ddot{\theta}y + \dot{\theta}^2 x - \frac{\mu}{r^3}x + A_x$$
$$\ddot{y} = -2\dot{\theta}\dot{x} - \ddot{\theta}x + \dot{\theta}^2 y + \frac{2\mu}{r^3}y + A_y$$
$$\ddot{r} = r\dot{\theta}^2 - \frac{\mu}{r^2}$$
$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r}$$

where r is the radial distance of the reference orbit from the sun, θ is the true anomaly, and μ is the gravitational parameter of the sun. Furthermore, we have

$$r = \frac{p}{1 + e\cos\theta}$$
$$\dot{r} = \sqrt{\mu/p}\,(e\sin\theta)$$
$$\dot{\theta} = \sqrt{\mu/p^3}\,(1 + e\cos\theta)^2$$

where $p = a(1 - e^2)$.

Consider the Option-3 SSGT case for Apophis with a = 0.92239 AU, e = 0.19, an assumed low-thrust acceleration of $A_x = 3.8284 \times 10^{-10}$ mm/s^2, $A_y = 5.4667 \times 10^{-10}$ mm/s^2, t_a = 5 years, and t_c = 3 years. The eccentricity effect on the non-secular terms is evident in Fig. 15, as compared to Fig. 14 for the same case but with $e = 0$.

To further examine the eccentricity effect, consider the Option 3 SSGT case for a 200-m asteroid (Fig. 2) with $M = 1.1 \times 10^{10}$ kg, a = 2.1537 AU, e = 0.6498, $A_x = 1.76 \times 10^{-9}$ mm/s^2, $A_y = 2.51 \times 10^{-9}$ mm/s^2.

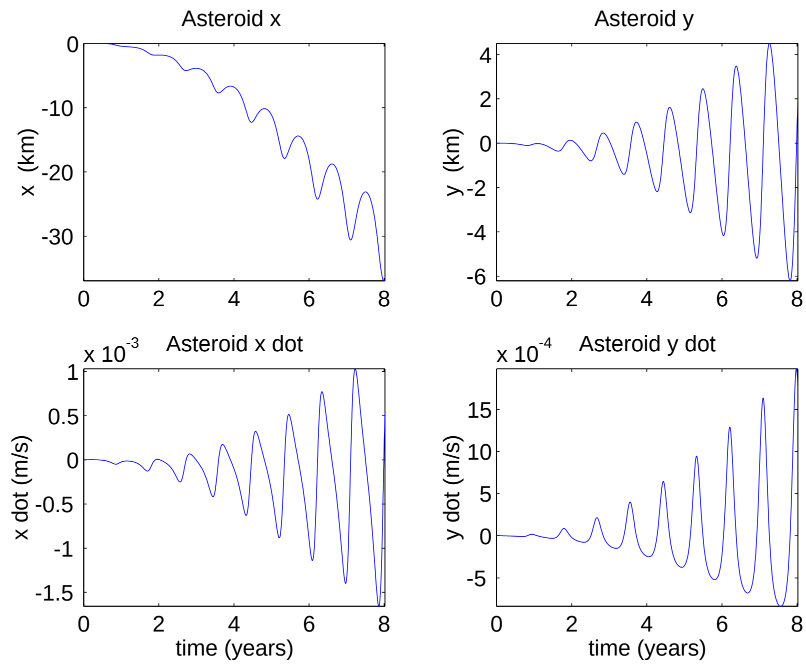

Figure 15. Long-term simulation of the SSGT for Apophis with e = 0.19.

Simulation results for t_a = 10 years and t_c = 12 years are shown in Figure 16. The significant effect of a large eccentricity (e = 0.6498) is evident in Figure 16. Consequently, further studies are needed to explore: (i) some recent research results for the eccentric C-W-H problem as described in Refs. 31-34 and (ii) a more realistic hovering control problem near asteroids as described in Refs. 35-36.

Appendix C: Gravity-Gradient Torque Estimation for Attitude Control

Consider the pitch-axis attitude dynamical model of the SSGT described by

$$I_2\ddot{\theta}_2 - \frac{3\mu}{R^3}(I_3 - I_1)\theta_2 = 0$$

where $\mu = GM = 3$ N-m^2/kg ($G = 6.6695 \times 10^{-11}$ N-m^2/kg^2, $M = 4.6 \times 10^{10}$ kg), $I_1 = I_2$ = 300,000 kg-m^2, I_3 = 600,000 kg-m^2, and R = 500 m. For this case, we have

$$I_2\ddot{\theta}_2 - 0.02\ \theta_2 = 0$$

The peak gravity-gradient torque can be estimated as 0.01 N-m for an assumed peak pitch/yaw attitude error of 30 deg. Such a small, peak gravity-gradient disturbance torque is not of practical concern because the SSGT nominally maintains a zero pitch/yaw pointing error.

References

[1]Belton, M., Morgan, T., Samarasinha, N., and Yeomans, D. (Eds.), *Mitigation of Hazardous Comets and Asteroids*, Cambridge University Press, 2005.

[2]Schweickart, R., Lu, E., Hut, P., and Chapman, C., "The Asteroid Tugboat," *Scientific American*, November 2003, pp. 54-61.

[3]Schweickart, R., Chapman, C., Durda, D., and Hut, P., "Threat Mitigation: The Asteroid Tugboat," White Paper No. 041, Presented at *NASA Workshop on NEO Detection, Characterization, and Threat Mitigation*, Vail, CO, June 26-29, 2006.

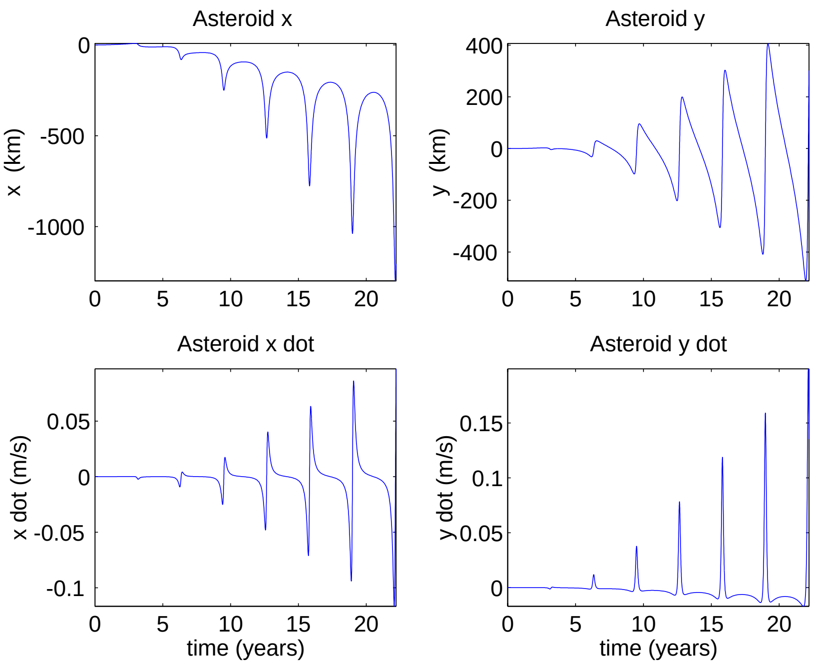

Figure 16. Long-term simulation of the SSGT for a 200-m asteroid with $M = 1.1 \times 10^{10}$ kg, a = 2.1537 AU, e = 0.6498, $A_x = 1.76 \times 10^{-9}$ mm/s^2, $A_y = 2.51 \times 10^{-9}$ mm/s^2, t_a = 10 years, and t_c = 12 years.

[4]Wright, J. L. *Space Sailing*, Gordon and Breach Science Publishers, Philadelphia, 1992.

[5]McInnes, C. R., *Solar Sailing: Technology, Dynamics and Mission Applications*, Springer Praxis Publishing, New York, 1999.

[6]McInnes, C. R., "Deflection of Near-Earth Asteroids by Kinetic Energy Impacts from Retrograde Orbits," *Planetary and Space Science*, Vol. 52, 2004, pp. 587-590.

[7]Wie, B., "Solar Sailing Kinetic Energy Interceptor Mission for Impacting and Deflecting Near-Earth Asteroids," AIAA-2005-3725, *41st AIAA Joint Propulsion Conference and Exhibit*, Tucson, AZ, July 10-13, 2005.

[8]Dachwald, B. and Wie, B., "Solar Sail Trajectory Optimization for Intercepting, Impacting, and Deflecting Near-Earth Asteroids," AIAA-2005-6176, *AIAA Guidance, Navigation, and Control Conference*, San Francisco, CA, August 15-18, 2005.

[9]Wie, B., "Solar Sailing Kinetic Energy Impactor Mission Design for Impacting and Deflecting Near-Earth Asteroids," White Paper No. 009, Presented at *NASA Workshop on NEO Detection, Characterization, and Threat Mitigation*, Vail, CO, June 26-29, 2006.

[10]Dachwald, B., Kahle, R., and Wie, B., "Solar Sailing KEI Mission Design Tradeoffs for Impacting and Deflecting Asteroid 99942 Apophis," AIAA-2006-6178, *AIAA/AAS Astrodynamics Specialists Conference*, Keystone, CO, August 21-24, 2006.

[11]Lu, E. and Love, S., "Gravitational Tractor for Towing Asteroids," *Nature*, Vol. 438, 10 November 2005, pp. 177-178.

[12]McInnes, C. R., "Near Earth Object Orbit Modification Using Gravitational Coupling," to appear in *Journal of Guidance, Control, and Dynamics*, 2007.

[13]Shkadov. L. M., "Possibility of Controlling Solar System Motion in the Galaxy, Paper IAA-87-613, *38th International Astronautical Federation Congress*, Brighton, UK, October 10-17, 1987.

[14]McInnes, C. R., "Astronomical Engineering Revisited: Planetary Orbit Modification Using Solar Radiation Pressure," *Astrophysics and Space Science*, Vol. 282, No. 4, 2002, pp. 765-772.

[15]Gennery, D. B., "What Should Be Done About Asteroid Apophis (2004 MN4)?", August 7, 2005.
http://www.spaceref.com/news/viewsr.html?pid=17666

[16]Kahle, R., Hahn, G., and Kührt, E., "Optimal Deflection of NEOs in Route of Collision with the Earth," Icarus, Vol. 182, Issue 2, June 2006, pp. 482-488.

[17]Chodas, P. and Yeomans, D., "Predicting Close Approaches and Estimating Impact Probabilities for Near-Earth Objects," AAS 99-462, *AAS/AIAA Astrodynamics Specialists Conference*, Girdwood, Alaska, August 16-19, 1999.

[18]Valsecchi, G., Milani, A., Rossi, A., and Tommei, G., "2004 MN4 Keyholes," *IAU Symposium No. 229: Asteroids, Comets, Meteors*, Rio de Janeiro, Brazil, August 7-12, 2005.

[19]Chesley, S. R.,"Potential Impact Detection for Near-Earth Asteroids: The Case of 99942 Apophis (2004 MN4)," *IAU Symposium No. 229: Asteroids, Comets, Meteors*, Rio de Janeiro, Brazil, August 7-12, 2005.

[20]Junkins, J., Singla, P., and Davis, J., "Impact Keyholes and Collision Probability Analysis for Resonant Encounter Asteroids," White Paper No. 070, Presented at *NASA Workshop on NEO Detection, Characterization, and Threat Mitigation*, Vail, CO, June 26-29, 2006.

[21]Wie, B., *Space Vehicle Dynamics and Control*, AIAA Education Series, AIAA, 1998.

[22]Scheeres, D., Williams, B., and Miller, J., "Evaluation of the Dynamic Environment of an Asteroid: Applications to 433 Eros," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 3, 2000, pp. 466-475.

[23]Hu, W. and Scheeres, D., "Spacecraft Motion about Slowly Rotating Asteroids," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 4, 2002, pp. 765-775.

[24]Garbe, G. and Montgomery, E., "An Overview of NASA's Solar Sail Propulsion Project," AIAA-2003-4662, *39th AIAA Joint Propulsion Conference and Exhibit*, Huntsville, AL, July 20-23, 2003.

[25]Murphy, D. M., McEachen, M. E., Macy, B. D., and Gaspar, J. L., "Demonstration of a 20-m Solar Sail System," AIAA-2005-2126, *46th Structures, Structural Dynamics, and Materials Conference*, Austin TX, April 18-21, 2005.

[26]Lichodziejewski, D., Derbès, B., Slade, K., and Mann, T., "Vacuum Deployment and Testing of a 4-Quadrant Scalable Inflatable Rigidizable Solar Sail System," AIAA-2005-3927, *41st AIAA Joint Propulsion Conference and Exhibit*, Tucson, AZ, July 10-13, 2005.

[27]Special Section on Solar Sails, *Journal of Spacecraft and Rockets*, to be published in 2007.

[28]Scheeres, D. J. and Schweickart, R. L., "The Mechanics of Moving Asteroids," AIAA-2004-1440, *2004 Planetary Defense Conference: Protecting Earth from Asteroids*, Garden Grove, CA, Feb. 23-26, 2004.

[29]Izzo, D., "On the Deflection of Potentially Hazardous Objects," AAS-05-141, *15th AAS/AIAA Space Flight Mechanics Conference*, Copper Mountain, CO, January 23-27, 2005.

[30]Izzo, D., Bourdoux, A., Walker, R., and Ongaro, F., "Optimal Trajectories for the Impulsive Deflection of Near Earth Objects," Paper IAC-05-C1.5.06, *56th International Astronautical Congress*, Fukuoka, Japan, October 17-21, 2005.

[31]Inalhan, G., Tillerson, M., and How, J.,"Relative Dynamics and Control of Spacecraft Formations in Eccentric Orbits," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 1, 2002, pp. 48-59.

[32]Yamanak, K. and Ankersen, F.,"New State Transition Matrix for Relative Motion on an Arbitrary Elliptical Orbit," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 1, 2002, pp. 60-66.

[33]Vaddi, S. S,, Vadali, S. R., and Alfriend, K. T.,"Formation Flying: Accommodating Nonlinearity and Eccentricity Perturbations," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 2, 2003, pp. 214-223.

[34]Alfriend, K. T. and Yan, H.,"Evaluation and Comparison of Relative Motion Theories," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 2, 2005, pp. 254-261.

[35]Broschart, S. B. and Scheeres, D. J., "Control of Hovering Spacecraft Near Small Bodies: Application to Asteroid 25142 Itokawa," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 2, 2005, pp. 343-354.

[36]Kawaguchi, J., "Hayabusa: Summary of Guidance, Navigation and Control Achievement in Its Proximity Phase," AIAA-2006-6533, *AIAA/AAS Astrodynamics Specialists Conference*, Keystone, CO, August 21-24, 2006.